

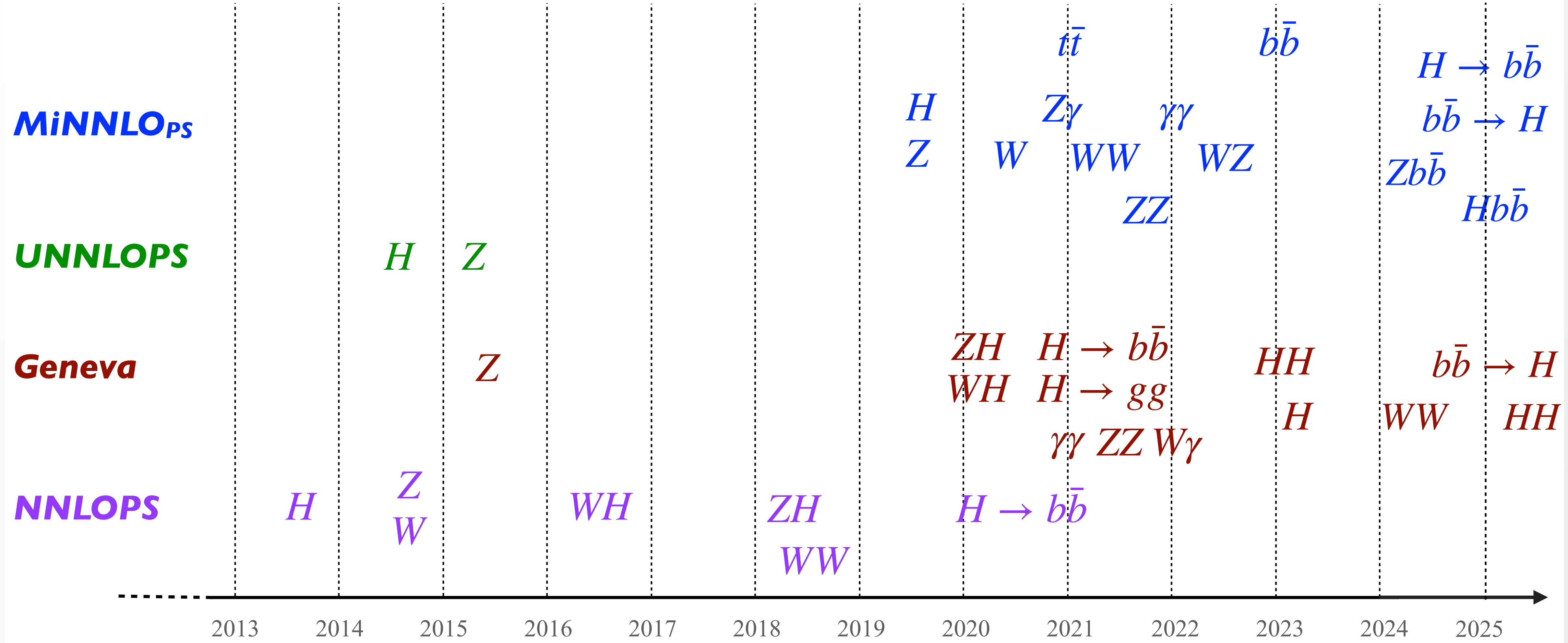
# Current status of NNLO+PS simulations at LHC

Luca Rottoli



# NNLO+PS timeline

Wiesemann @ SM@LHC 2023



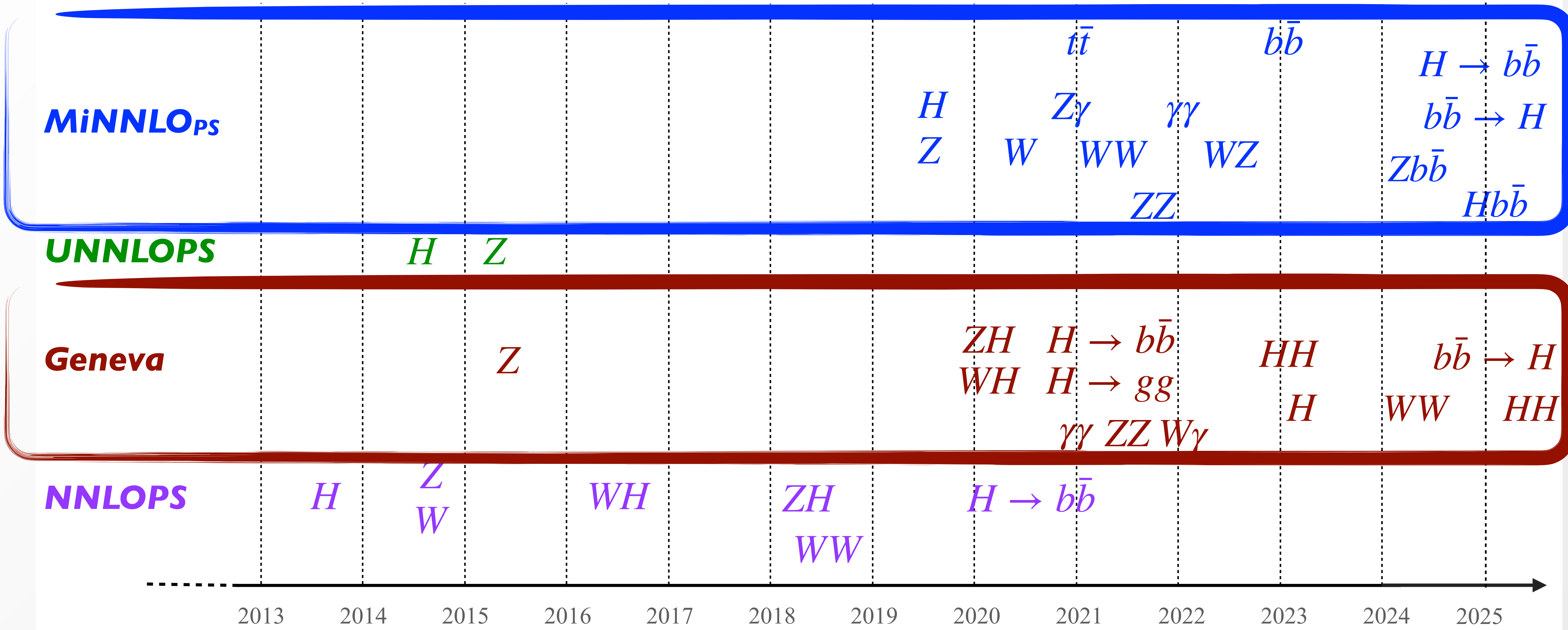
[Campbell, Höche, Li, Preuss, Skands '21]

[El-Menoufi, Preuss, Scyboz, Skands '24]

Progress in NNLO+PS with sector showers

# NNLO+PS timeline

Wiesemann @ SM@LHC 2023



Progress in NNLO+PS with sector showers  
 [Campbell, Höche, Li, Preuss, Skands '21]  
 [El-Menoufi, Preuss, Scyboz, Skands '24]

# NNLO+PS: general strategy

Recast perturbative NNLO calculation in a Monte Carlo language (radiation ordered in a given resolution variable)

- Introduce a **set of resolution variables** to measure hardness of first, second... emission
- Logarithmic dependence on resolution parameters **resummed explicitly** or via **Sudakov form factors**
- Fix remaining degrees of freedom by **matching** to NNLO computation (exploiting **resummation properties** of resolution variable)

# NNLO+PS: GENEVA vs. MiNNLO<sub>PS</sub>

GENEVA and MiNNLO<sub>PS</sub> methods achieve NNLO+PS accuracy following the same general strategy, **with some important differences:**

## GENEVA

- Originally developed using **jettiness-like** observables ( $\mathcal{T}_0, \mathcal{T}_1$ )
- **High-accuracy resummation** of residual logarithmic dependence
- **Additive-like** matching to reach NNLO accuracy (jettiness/ $q_T$  subtraction)
- Control of the missing power corrections required for NNLO accuracy

## MiNNLO<sub>PS</sub>

- Originally developed using **transverse-momentum** observables
- **Sudakov factors** used to resum logarithmic dependence on resolution parameters
- **Multiplicative-like** matching to reach NNLO accuracy (also inspired by jettiness/ $q_T$  subtraction)
- Absence of missing power corrections traded with terms beyond NNLO accuracy

# NNLO+PS: GENEVA vs. MiNNLO<sub>PS</sub>

GENEVA and MiNNLO<sub>PS</sub> methods achieve NNLO+PS accuracy following the same general strategy, **with some important differences:**

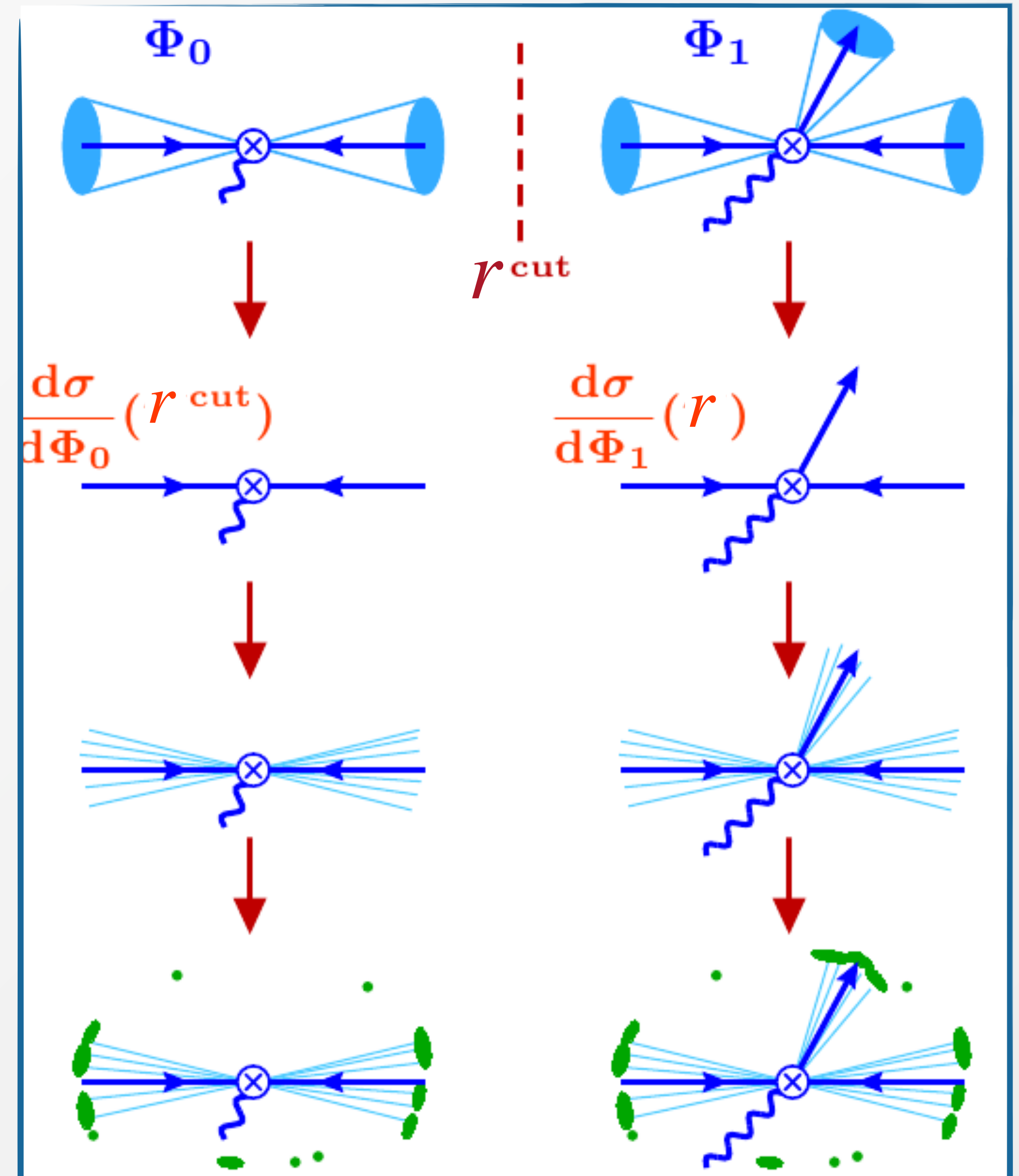
## GENEVA

- Originally developed using **jettiness-like** observables ( $\mathcal{T}_0, \mathcal{T}_1$ )
- **High-accuracy resummation** of residual logarithmic dependence
- **Additive-like** matching to reach NNLO accuracy (jettiness/ $q_T$  subtraction)
- Control of the missing power corrections required for NNLO accuracy

# GENEVA method in a nutshell

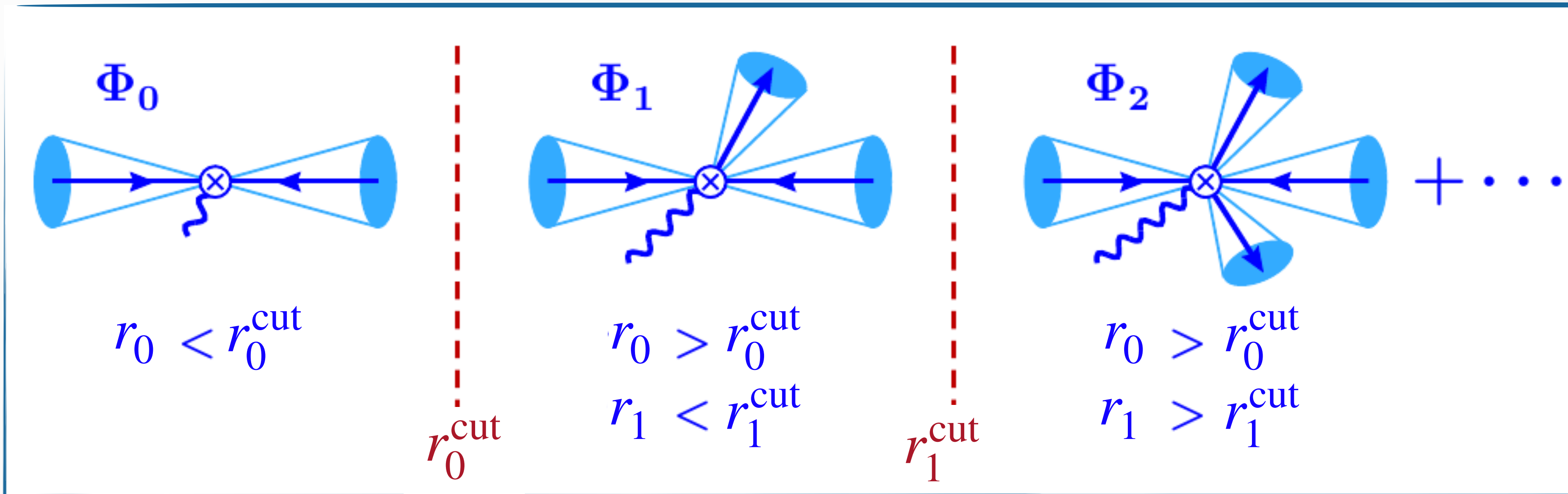
[Alioli, Bauer, Berggren, Tackmann, Walsh, Zuberi '15]

- Design IR-finite definition of events, based on **resolution parameter**  $r^{\text{cut}}$ . Emissions below  $r^{\text{cut}}$  are **unresolved** and the kinematic configuration considered is the one of the event before the emission
- Associate differential cross-sections to events such that 0-jet events are NNLO accurate and  $r$  is resummed at NNLL'
- Shower events
- Hadronise, add multi-parton interactions (MPI) and compare with data



# GENEVA method in a nutshell

Procedure can be iterated, thus slicing the phase space into jet-bins



Exclusive 0-jet bin

$$\frac{d\sigma_0^{\text{MC}}}{d\Phi_0}(r_0^{\text{cut}})$$

Exclusive 1-jet bin

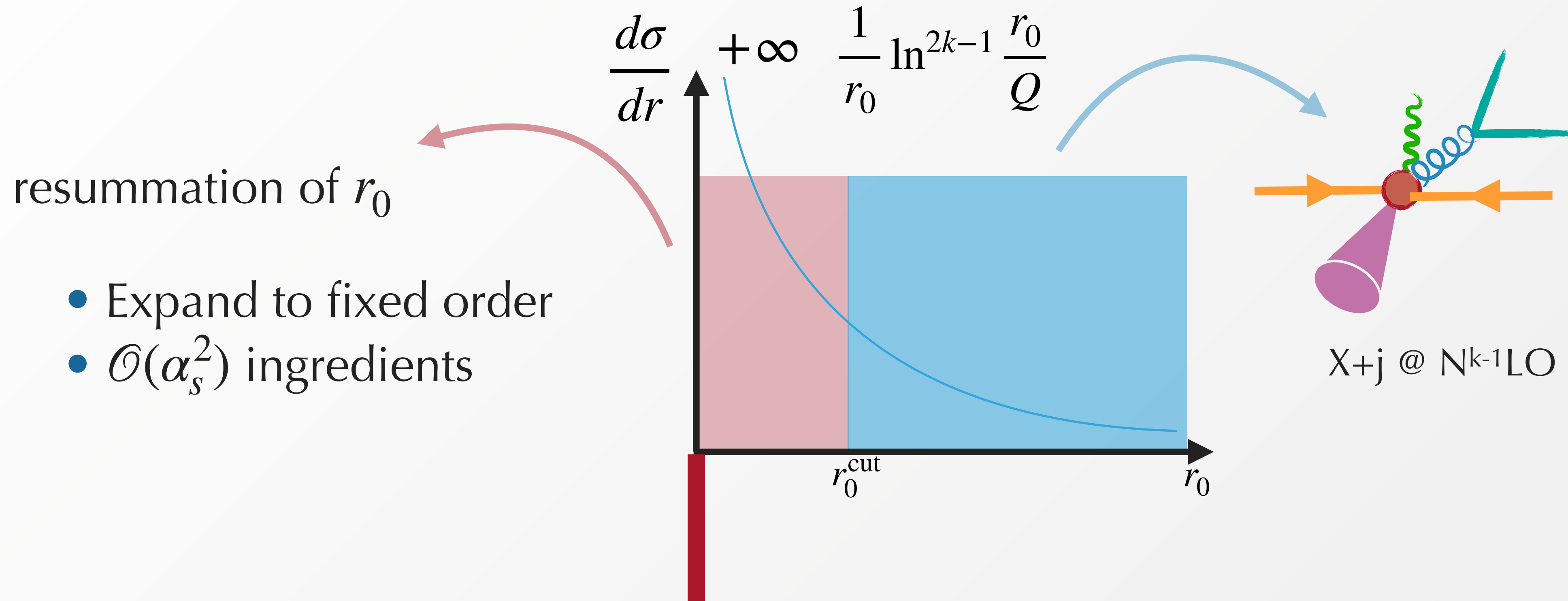
$$\frac{d\sigma_1^{\text{MC}}}{d\Phi_1}(r_0 > r_0^{\text{cut}}, r_1^{\text{cut}})$$

Inclusive 2-jet bin

$$\frac{d\sigma_2^{\text{MC}}}{d\Phi_2}(r_0 > r_0^{\text{cut}}, r_1 > r_1^{\text{cut}})$$

# GENEVA method in a nutshell: resummation of the resolution parameter

As we take  $r_0^{\text{cut}} \rightarrow 0$ , large logarithms of  $r_0^{\text{cut}}, r_0$  appear, which must be resummed lest they spoil the perturbative convergence



$$d\sigma_F^{\text{MC}} = d\sigma_F^{\text{res}} + [d\sigma_{FJ}]_{\text{f.o.}} - [d\sigma_F^{\text{res}}]_{\text{f.o.}}$$

NNLO accuracy guaranteed up to power correction in  $r_0^{\text{cut}}$

# GENEVA method in a nutshell: resummation of the resolution parameter

$$d\sigma_F^{\text{MC}} = d\sigma_F^{\text{res}} + [d\sigma_{FJ}]_{\text{f.o.}} - [d\sigma_F^{\text{res}}]_{\text{f.o.}}$$

Above formula can be compared to the  $q_T$  or **jettiness subtraction formalism**

[Catani, Grazzini '08][Gaunt, Stahlhofen, Tackmann, Walsh '15]

However, we are interested in a **fully differential Monte Carlo event generator**. Since the resummed component is only differential in Born phase space  $\Phi_0$  and  $r_0$ , one has to make it differential in 2 more variables, e.g. energy ratio  $z = E_m/E_s$ , azimuthal angle  $\phi$ .

$$\frac{d\sigma_{FJ}^{\text{MC}}}{d\Phi_{FJ}}(r_0 > r_0^{\text{cut}}) = \frac{d\sigma^{\text{res}}}{d\Phi_F dr_0} \mathcal{P}(\Phi_{FJ}) + \frac{d\sigma^{\text{NLO}_{FJ}}}{d\Phi_{FJ}} - \left[ \frac{d\sigma^{\text{res}}}{d\Phi_F dr_0} \mathcal{P}(\Phi_{FJ}) \right]_{\text{NLO}}$$

Here  $\mathcal{P}(\Phi_{FJ})$  is a normalised splitting probability to make the resummation differential in  $\Phi_{FJ}$

$$\int \frac{d\Phi_{FJ}}{d\Phi_{FJ} dr_0} \mathcal{P}(\Phi_{FJ}) = 1$$

# GENEVA method in a nutshell: 1-/2-jet separation

An analogue separation is performed for the 1-jet cross section, which is partitioned into an exclusive 1-jet cross section and an inclusive 2-jet cross section

$$d\sigma_{FJ}^{\text{MC}} = d\sigma_{FJ}^{\text{res}} + [d\sigma_{FJJ}]_{\text{f.o.}} - [d\sigma_{FJ}^{\text{res}}]_{\text{f.o.}}$$

Integrated quantities retain NLO accuracy via **local subtraction**; resummation accuracy at NLL is sufficient

Analogously to the 0-/1-jet separation, a **normalised splitting function**  $\mathcal{P}(\Phi_{FJJ})$  is needed to make the extend the differential dependence of  $d\sigma_{FJ}^{\text{res}}$

$$U_1(\Phi_{FJ}, r_1^{\text{cut}}) + \int \frac{d\Phi_{FJJ}}{d\Phi_{FJ}} U_1'(\Phi_{FJ}, r_1) \mathcal{P}(\Phi_{FJJ}) \theta(r_1 > r_1^{\text{cut}}) = 1$$



Sudakov form factor resumming  $r_1$  dependence

# NNLO+PS: GENEVA vs. MiNNLO<sub>PS</sub>

GENEVA and MiNNLO<sub>PS</sub> methods achieve NNLO+PS accuracy following the same general strategy, **with some important differences:**

## MiNNLO<sub>PS</sub>

- Originally developed using **transverse-momentum** observables
- **Sudakov factors** used to resum logarithmic dependence on resolution parameters
- **Multiplicative-like** matching to reach NNLO accuracy (also inspired by jettiness/ $q_T$  subtraction)
- Absence of missing power corrections traded with terms beyond NNLO accuracy

## MiNNLO<sub>PS</sub> in a nutshell [Monni, Nason, Re, Wiesemann, Zanderighi '19]

Starting point of MiNNLO<sub>PS</sub> construction is analogue to the formulae above

$$d\sigma^F = d\sigma_F^{\text{res}} + [d\sigma_{FJ}]_{\text{f.o.}} - [d\sigma_F^{\text{res}}]_{\text{f.o.}}$$

Up to the second perturbative order, the resummed component can be written as a total derivative

$$\frac{d\sigma_F^{\text{res}}}{dp_T d\Phi_B} = \frac{d}{dp_T} \{ e^{-S} \mathcal{L} \} = e^{-S} \underbrace{\{ S' \mathcal{L} + \mathcal{L}' \}}_{\equiv D}$$

where the luminosity  $\mathcal{L}$  and the Sudakov form factor  $S$  are written in terms of the ingredients of  $q_T$  resummation at N<sup>3</sup>LL accuracy

$$\mathcal{L} \sim H(C \otimes f)(C \otimes f) \quad S(p_T) = \int_{p_T}^Q \frac{dq}{q} \left( A(\alpha_s(q)) \ln \frac{Q^2}{q^2} + B(\alpha_s(q)) \right)$$

# MiNNLO<sub>PS</sub> in a nutshell

By factoring out the Sudakov exponential factor

$$d\sigma^F = d\sigma_F^{\text{res}} + [d\sigma_{FJ}]_{\text{f.o.}} - [d\sigma_F^{\text{res}}]_{\text{f.o.}} = e^{-S} \left\{ D + \frac{[d\sigma_{FJ}]_{\text{f.o.}}}{[e^{-S}]_{\text{f.o.}}} - \frac{[d\sigma_F^{\text{res}}]_{\text{f.o.}}}{[e^{-S}]_{\text{f.o.}}} \right\}$$

Expanding up to  $\mathcal{O}(\alpha_s^3(p_T))$  one gets

$$\begin{aligned} \frac{d\sigma_F^{\text{MiNNLO}}}{dp_T d\Phi_B} = e^{-S(p_T)} & \left\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi} \frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T))} \left( 1 + \frac{\alpha_s}{2\pi} S^{(1)}(p_T) \right) + \underbrace{\left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 \frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T^2))} \right. \\ & \left. + \underbrace{\left[ D(p_T) - \frac{\alpha_s}{2\pi} D^{(1)}(p_T) - \left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 D^{(2)}(p_T) \right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3) \right\} \end{aligned}$$

# MiNNLO<sub>PS</sub> in a nutshell

First line equivalent to the **MiNLO'** formulation



$$\frac{d\sigma_F^{\text{MiNNLO}}}{dp_T d\Phi_B} = e^{-S(p_T)} \left\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi} \frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T))} \left( 1 + \underbrace{\frac{\alpha_s}{2\pi} S^{(1)}(p_T)}_{\mathcal{O}(\alpha_s(p_T))} \right) + \underbrace{\left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 \frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T^2))} \right. \\ \left. + \underbrace{\left[ D(p_T) - \frac{\alpha_s}{2\pi} D^{(1)}(p_T) - \left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 D^{(2)}(p_T) \right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3) \right\}$$

# MiNNLO<sub>PS</sub> in a nutshell

Second lines contains the additional terms needed to reach **NNLO accuracy**, upon integration in  $q_T$

$$\begin{aligned}
 \frac{d\sigma_F^{\text{MiNNLO}}}{dp_T d\Phi_B} = & e^{-S(p_T)} \left\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi} \frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T))} \left( 1 + \underbrace{\frac{\alpha_s}{2\pi} S^{(1)}(p_T)}_{\mathcal{O}(\alpha_s(p_T^2))} \right) + \underbrace{\left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 \frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T^2))} \right. \\
 & \left. + \underbrace{\left[ D(p_T) - \frac{\alpha_s}{2\pi} D^{(1)}(p_T) - \left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 D^{(2)}(p_T) \right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3) \right\}
 \end{aligned}$$

# MiNNLO<sub>PS</sub> in a nutshell

Regular terms contribute **beyond NNLO accuracy**

$$\begin{aligned}
 \frac{d\sigma_F^{\text{MiNNLO}}}{dp_T d\Phi_B} = & e^{-S(p_T)} \left\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi} \frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T))} \left( 1 + \underbrace{\frac{\alpha_s}{2\pi} S^{(1)}(p_T)}_{\mathcal{O}(\alpha_s(p_T^2))} \right) + \underbrace{\left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 \frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T^2))} \right. \\
 & \left. + \underbrace{\left[ D(p_T) - \frac{\alpha_s}{2\pi} D^{(1)}(p_T) - \left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 D^{(2)}(p_T) \right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3) \right\}
 \end{aligned}$$

# MiNNLO<sub>PS</sub> in a nutshell

**NNLO subtraction** accomplished thanks to the presence of a **Sudakov form factor** which exponentially suppressed the  $q_T \rightarrow 0$  limit

$$\begin{aligned}
 \frac{d\sigma_F^{\text{MiNNLO}}}{dp_T d\Phi_B} = & \underbrace{e^{-S(p_T)}}_{\text{Sudakov form factor}} \left\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi} \frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B} \left( 1 + \frac{\alpha_s}{2\pi} S^{(1)}(p_T) \right)}_{\mathcal{O}(\alpha_s(p_T))} + \underbrace{\left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 \frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T)^2)} \right. \\
 & \left. + \underbrace{\left[ D(p_T) - \frac{\alpha_s}{2\pi} D^{(1)}(p_T) - \left( \frac{\alpha_s(p_T)}{2\pi} \right)^2 D^{(2)}(p_T) \right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3) \right\}
 \end{aligned}$$

## MiNNLO<sub>PS</sub> in a nutshell

NNLO+PS construction achieved by applying the above formulae to the POWHEG calculation for F+j production, making it NNLO accurate

$$\frac{d\sigma}{d\Phi_{FJ}} = \tilde{B}^{FJ} \times \left\{ \Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + \int d\Phi_{\text{rad}} \Delta(p_{T,\text{rad}}) \frac{R(\Phi_{FJ}, \Phi_{\text{rad}})}{B(\Phi_{FJ})} \right\}$$



$$\frac{d\sigma}{d\Phi_{FJ}} = \tilde{B}_{FJ}^{\text{MiNNLO}_{\text{PS}}} \times \left\{ \Delta_{\text{pwg}}(\Lambda_{\text{pwg}}) + \int d\Phi_{\text{rad}} \Delta(p_{T,\text{rad}}) \frac{R(\Phi_{FJ}, \Phi_{\text{rad}})}{B(\Phi_{FJ})} \right\}$$

# MiNNLO<sub>PS</sub> in a nutshell

NNLO+PS construction achieved by applying the above formulae to the POWHEG calculation for F+j production, making it NNLO accurate

$$\tilde{B}^{FJ} \sim \left\{ d\sigma_{FJ}^{(1)} + d\sigma_{FJ}^{(2)} \right\}$$



$$\tilde{B}^{\text{MiNNLOPS}}(\Phi_{FJ}) \simeq e^{-S(p_T)} \left\{ \frac{\alpha_s}{2\pi} \left[ \frac{d\sigma}{d\Phi_{FJ}} \right]^{(1)} \left( 1 + \frac{\alpha_s}{2\pi} [S(p_T)]^{(1)} \right) + \left( \frac{\alpha_s}{2\pi} \right)^2 \left[ \frac{d\sigma}{d\Phi_{FJ}} \right]^{(2)} + (D(p_T) - D^{(1)}(p_T) - D^{(2)}(p_T)) \times \mathcal{P}(\Phi_{FJ}) \right\}$$

Here  $\mathcal{P}(\Phi_{FJ})$  again is needed to spread the last term in the  $\Phi_{FJ}$  phase space

# Choice of the resolution parameter (1)

**Any resolution variable** which can be **resummed at high enough accuracy** can be used in the formulation of a NNLO+PS method

Although initially formulated with a specific variable in mind, both GENEVA and MiNNLOPS have been (and can) be extended to other variables

The **availability of different resolution variables** within the same formalism allows one to study the **robustness of the frameworks** and **assess the uncertainties** associated to the choice of the resolution parameter, but...

# Choice of the resolution parameter (1)

**Any resolution variable** which can be **resummed at high enough accuracy** can be used in the formulation of a NNLO+PS method

Although initially formulated with a specific variable in mind, both GENEVA and MiNNLOPS have been (and can) be extended to other variables

The **availability of different resolution variables** within the same formalism allows one to study the **robustness of the frameworks** and **assess the uncertainties** associated to the choice of the resolution parameter, but...

...unfortunately, such comparisons have not been performed systematically.

As we will discuss later, the choice of the resolution parameters has important implications on a variety of aspects in the formulation of a NNLO+PS method

## Extension(s) of the GENEVA framework

The GENEVA method was formulated in **full generality**, making its extension formally viable

**Technically challenging** as it requires acting on all aspects of the framework (interplay with resummation, subtractions, mapping, shower interface...)

Availability of N<sup>3</sup>LL resummation for  $q_T$ :

$$\mathcal{T}_0 \rightarrow q_T$$

[Alioli, Bauer, Broggio,  
Gavardi, Kallweit, Lim,  
Nagar, Napoletano, LR '21]

[Gavardi, Lim, Von Kuck '25]

Recently extended to use also the leading jet  $p_T$  as resolution variable

$$\mathcal{T}_0 \rightarrow p_T^{j_1}$$

[Gavardi, Lim, Alioli, Tackmann '23]

thanks to the availability of NNLL' ingredients for  $p_T^{j_1}$

[Abreu, Gaunt, Monni, LR, Szafron '22]

Recent works also acted on the 1/2 jet separation, moving from SCET<sub>I</sub> to SCET<sub>II</sub> variables using either  $p_T^{j_2}$  or a  $k_T$ -like version of 1-jettiness  $\mathcal{T}_1$

[Gavardi, Lim, Alioli, Tackmann '23] [Gavardi, Lim, Von Kuck '25]

# Extension of the MiNNLO<sub>PS</sub> framework

Extension of the MiNNLO<sub>PS</sub> formalism to other (SCET<sub>I</sub>) resolution variables **less straightforward**, due to the connection with the transverse resummation formalism

Differences with respect to the transverse momentum case arise from the **different singular structure** (SCET<sub>I</sub> vs SCET<sub>II</sub>) which leads to a richer structure up to order  $\alpha_s^2$

$$\frac{d\sigma^{\text{sing}}(\mathcal{T}_0)}{d\Phi_B} = e^{-\mathcal{S}(\mathcal{T}_0)} \left[ \mathcal{L}(\mathcal{T}_0) \left( 1 - \frac{\zeta_2}{2} [(\mathcal{S}')^2 - \mathcal{S}'' ] - \zeta_3 \mathcal{S}' \mathcal{S}'' + \frac{3\zeta_4}{16} (\mathcal{S}'')^2 + \frac{\zeta_3}{3} \mathcal{S}''' \right) \right. \\ \left. + \mathcal{L}'(\mathcal{T}_0) (\zeta_2 \mathcal{S}' + \zeta_3 \mathcal{S}'') - \frac{\zeta_2}{2} \mathcal{L}''(\mathcal{T}_0) + \mathcal{O}(\alpha_s^3) \right]$$

[Ebert, LR, Wiesemann, Zanderighi, Zanolini '23]

To be compared with

$$\frac{d\sigma^{\text{sing}}(p_T)}{d\Phi_B} = e^{-\mathcal{S}(p_T)} \left[ \mathcal{L}(p_T) \left( 1 - \frac{\zeta_3}{4} \mathcal{S}' \mathcal{S}'' + \frac{\zeta_3}{12} \mathcal{S}''' \right) - \frac{\zeta_3}{4} \frac{\alpha_s(p_T)}{\pi} \mathcal{S}'' \hat{P} \otimes \mathcal{L}(p_T) + \mathcal{O}(\alpha_s^3) \right]$$

[Monni, Nason, Re, Wiesemann, Zanderighi '19]

# Extension of the MiNNLO<sub>PS</sub> framework

It turns out that the MiNNLO<sub>PS</sub> construction is **sufficiently flexible** to allow for its extension to a rather different variable such as jettiness, provided that the POWHEG calculation is modified in a suitable manner

$$q_T \rightarrow \mathcal{T}_0$$

With the POWHEG  $\tilde{B}$  function now reading

$$\tilde{B}^{\text{MiNNLOPS}}(\Phi_{FJ}) \simeq e^{-S(\mathcal{T}_0)} \left\{ \frac{\alpha_s}{2\pi} \left[ \frac{d\sigma}{d\Phi_{FJ}} \right]^{(1)} \left( 1 + \frac{\alpha_s}{2\pi} [S(\mathcal{T}_0)]^{(1)} \right) + \left( \frac{\alpha_s}{2\pi} \right)^2 \left[ \frac{d\sigma}{d\Phi_{FJ}} \right]^{(2)} + (D(\mathcal{T}_0) - D^{(1)}(\mathcal{T}_0) - D^{(2)}(\mathcal{T}_0)) \times \mathcal{P}(\Phi_{FJ}) \right\}$$

[Ebert, [LR](#), Wiesemann, Zanderighi, Zanolini '23]

## Choice of the resolution parameter (2)

The choice of resolution variables is **in principle immaterial** to reach NNLO accuracy

However, its choice has important consequences

- Size of **missing power corrections** (in the **GENEVA method**)
- Ease of **interface with the shower**
- Overall description of **physical events** after matching and showering

## Choice of the resolution parameter (2)

The choice of resolution variables is **in principle immaterial** to reach NNLO accuracy

However, its choice has important consequences

- Size of **missing power corrections** (in the **GENEVA method**)
- Ease of interface with the shower
- Overall description of **physical events** after matching and showering

# Resolution parameter and missing power corrections (1)

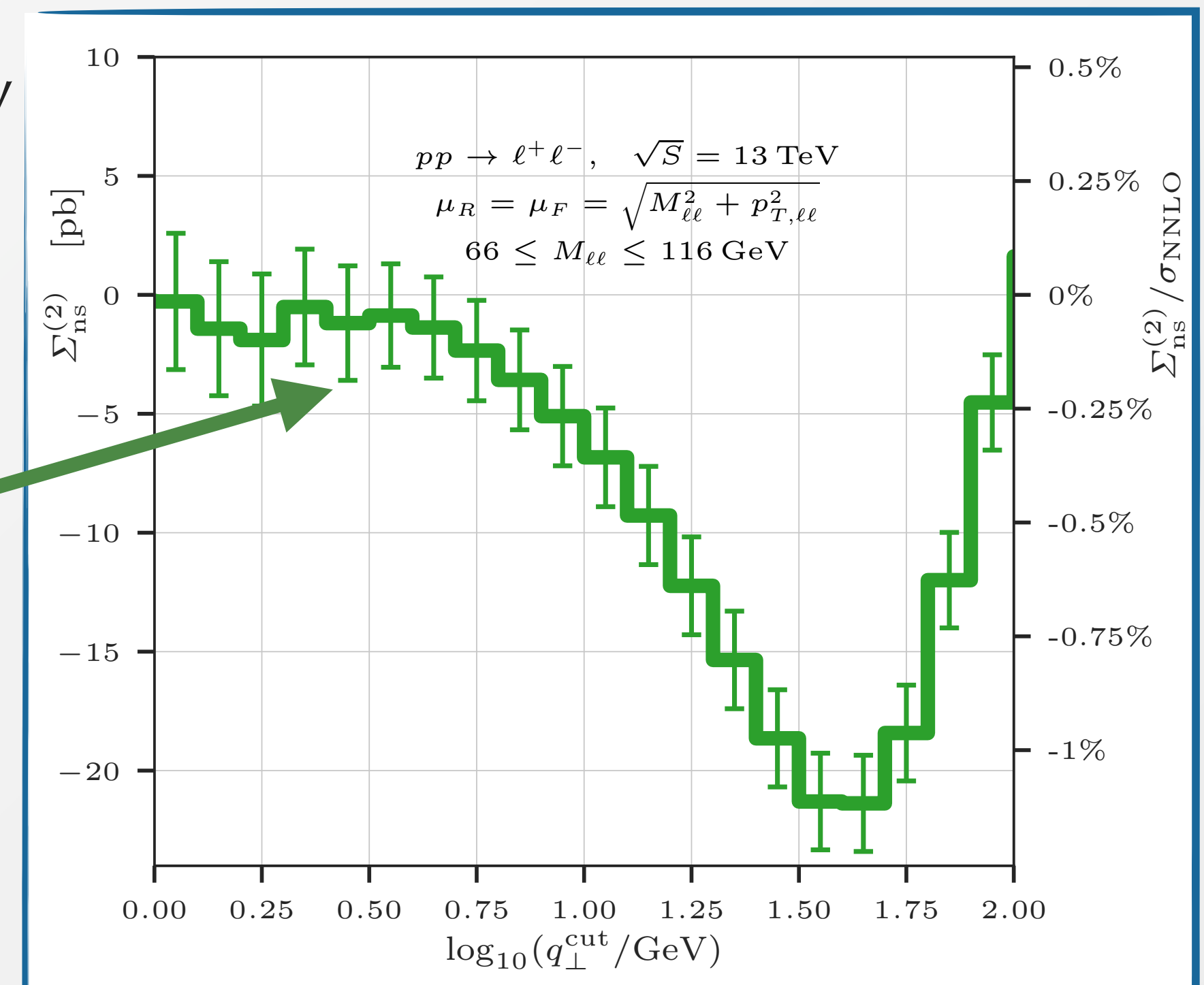
The GENEVA method relies on a **non-local subtraction scheme** to reach NNLO accuracy

As such, it is prone to the same limitation of non-local subtraction schemes, i.e. sensitivity to **missing power corrections** below the technical cutoff  $r_0^{\text{cut}}$

GENEVA $_{\mathcal{T}_0}$  typically relies on an overall reweighing of the events to reproduce the NNLO cross section due to larger missing power corrections using  $\mathcal{T}_0$

This drawback is removed **in the colour singlet case** by relying on transverse momentum observables  $(q_T, p_T^j)$

**Reduced size of power corrections** using transverse-momentum based observables removes the need of such reweighing improving comparisons with fixed-order computations



# Resolution parameter and missing power corrections (2)

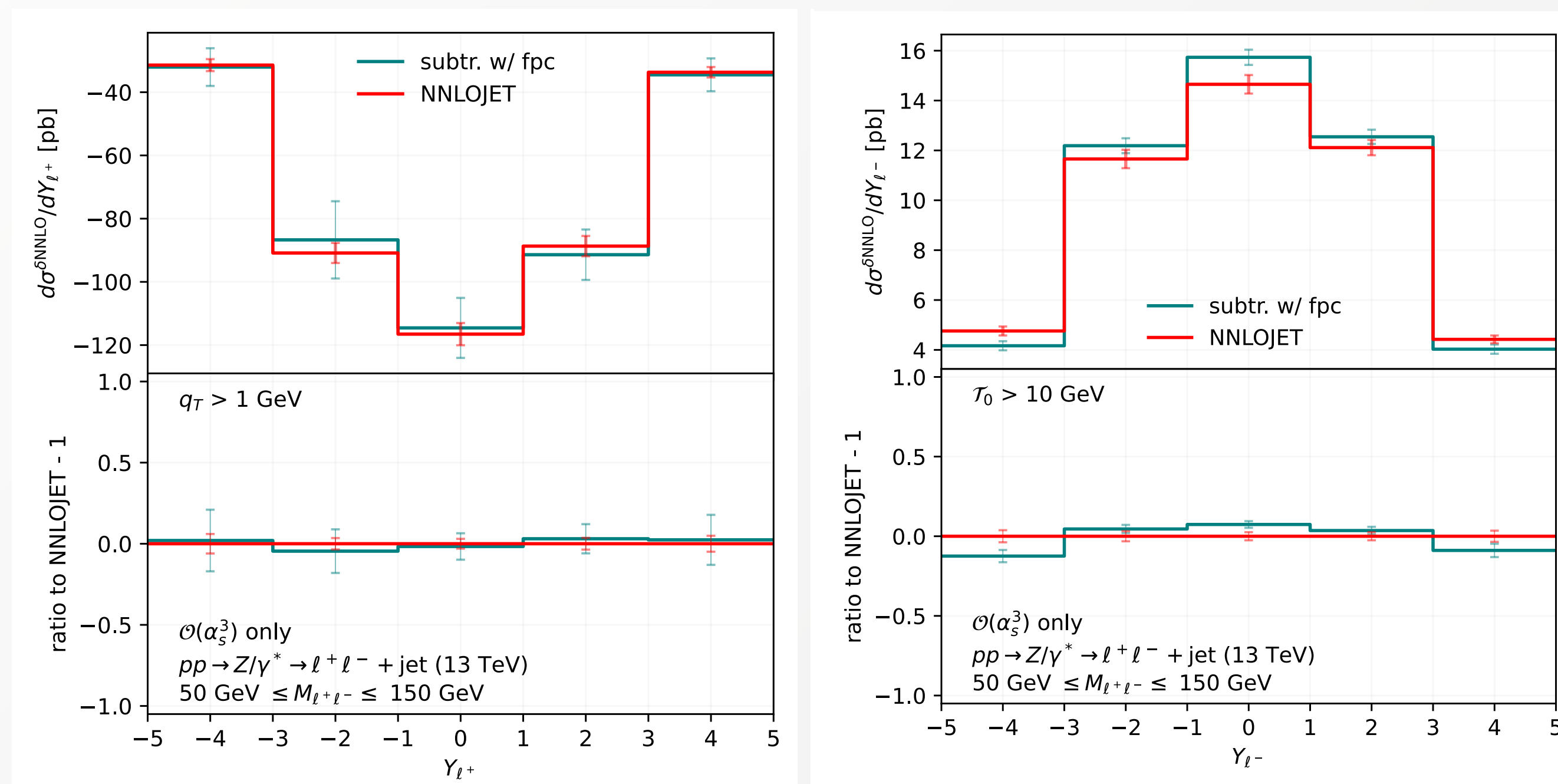
The treatment of power corrections remains delicate for more complex processes with multiple coloured legs at leading order (V+jet, heavy quark production).

All the observables studied and used so far for these processes display **linear, log-enhanced power corrections** at NNLO

Impact of missing power corrections can be reduced recurring to recoil prescription or using a projection-to-Born method and/or by subtracting the leading behaviour

[Campbell, Neumann, Vita '24]

Level of agreement between local and non-local subtraction depends on the fiducial cut



*See talk by A. Broggio*

Implementation within a NNLO+PS code may be not straightforward

[Alioli, Billis, Broggio, Stagnitto '25]

## Choice of the resolution parameter (2)

The choice of resolution variables is **in principle immaterial** to reach NNLO accuracy

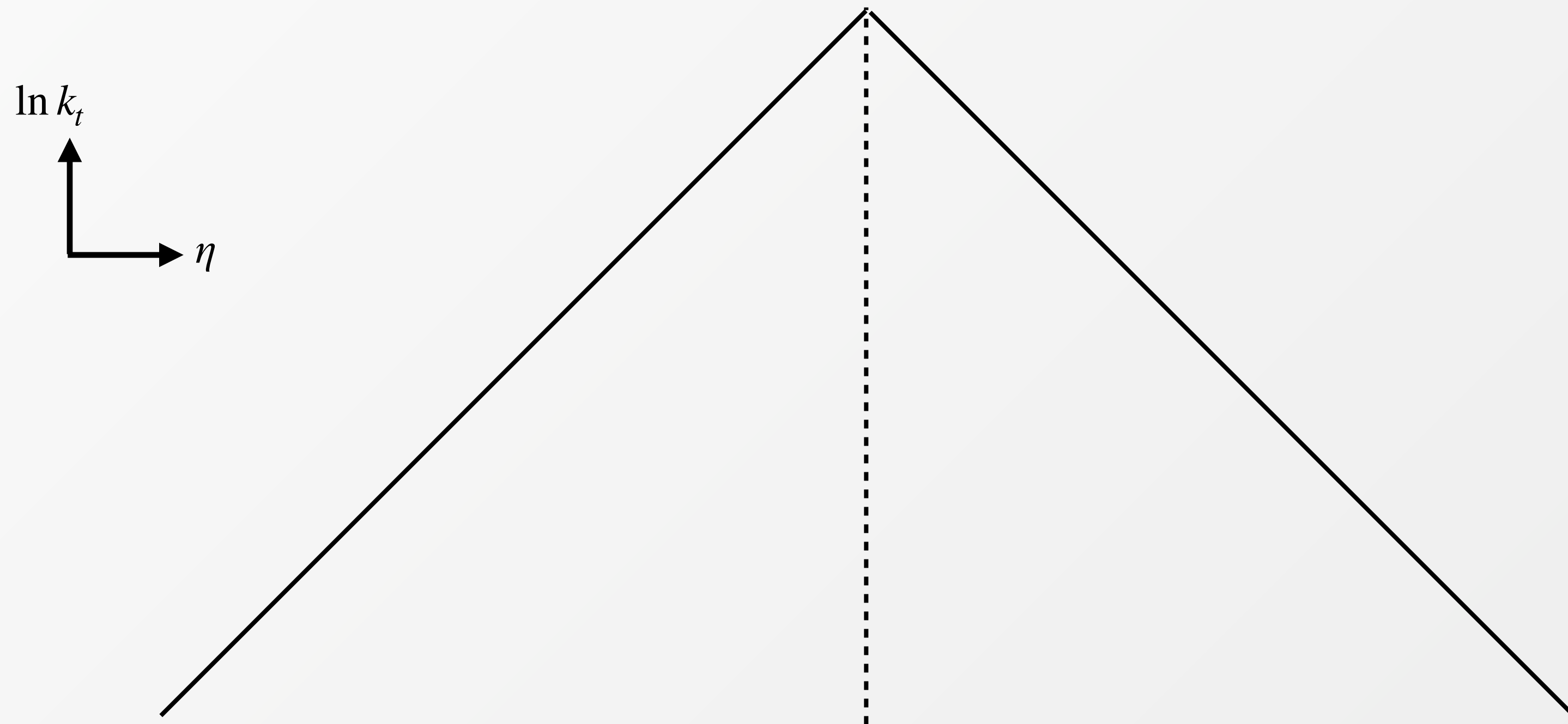
However, its choice has important consequences

- Size of missing power corrections **(in the GENEVA method)**
- Ease of **interface with the shower**
- Overall description of **physical events** after matching and showering

# Interplay with the parton shower

The interplay with the parton shower is likely the **most delicate aspect** of NNLO+PS methods

For simplicity, let's consider the interplay between the generator and the parton shower at NLO+PS using a Lund plane representation of the of the phase space for soft and/or collinear emissions

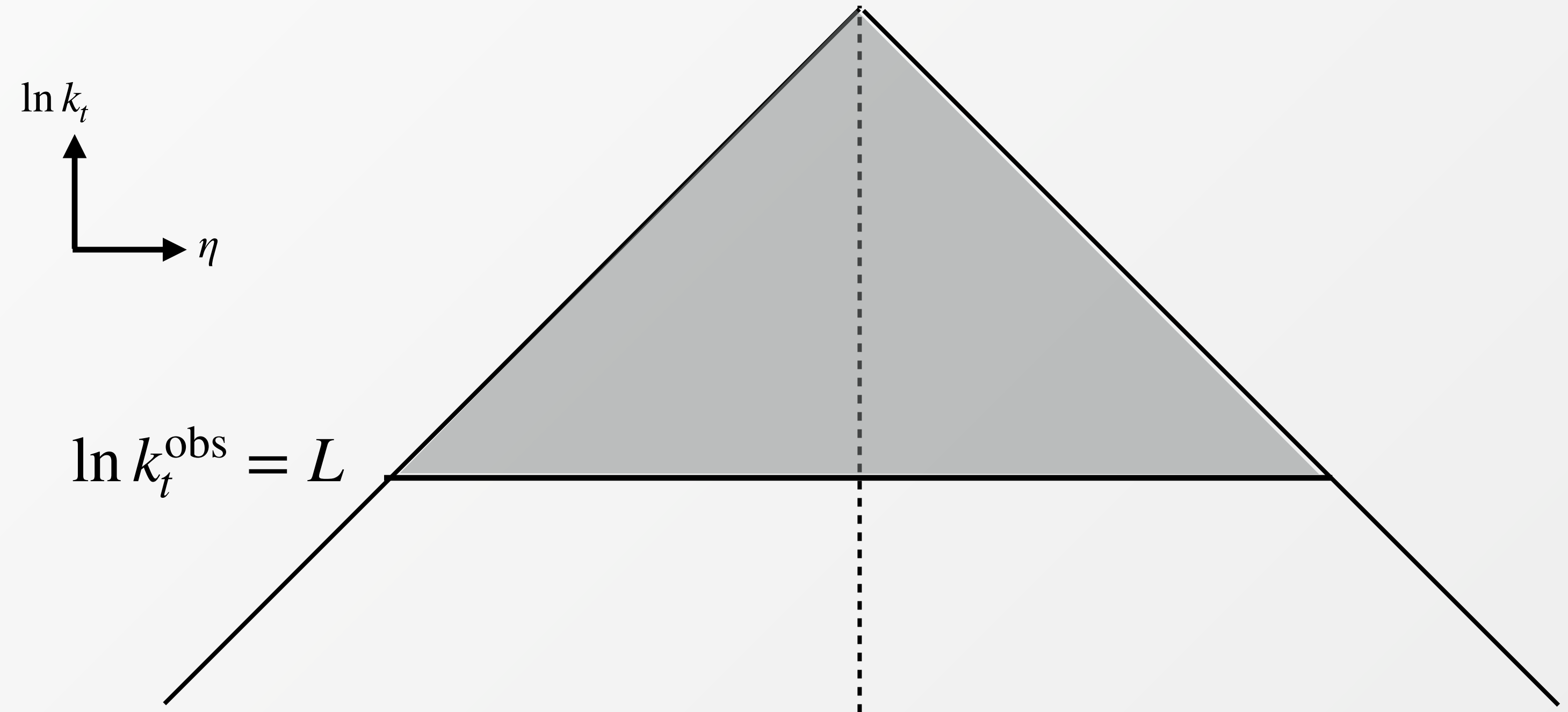


# Interplay with the parton shower

Let's assume to be interested in calculating the probability  $\Sigma(O < e^L)$  that an observable  $O$  is below a given threshold (here  $L < 0$ ).

Let us consider an observable which for a single soft collinear emission scales as

$$O \sim \frac{k_t}{Q}$$

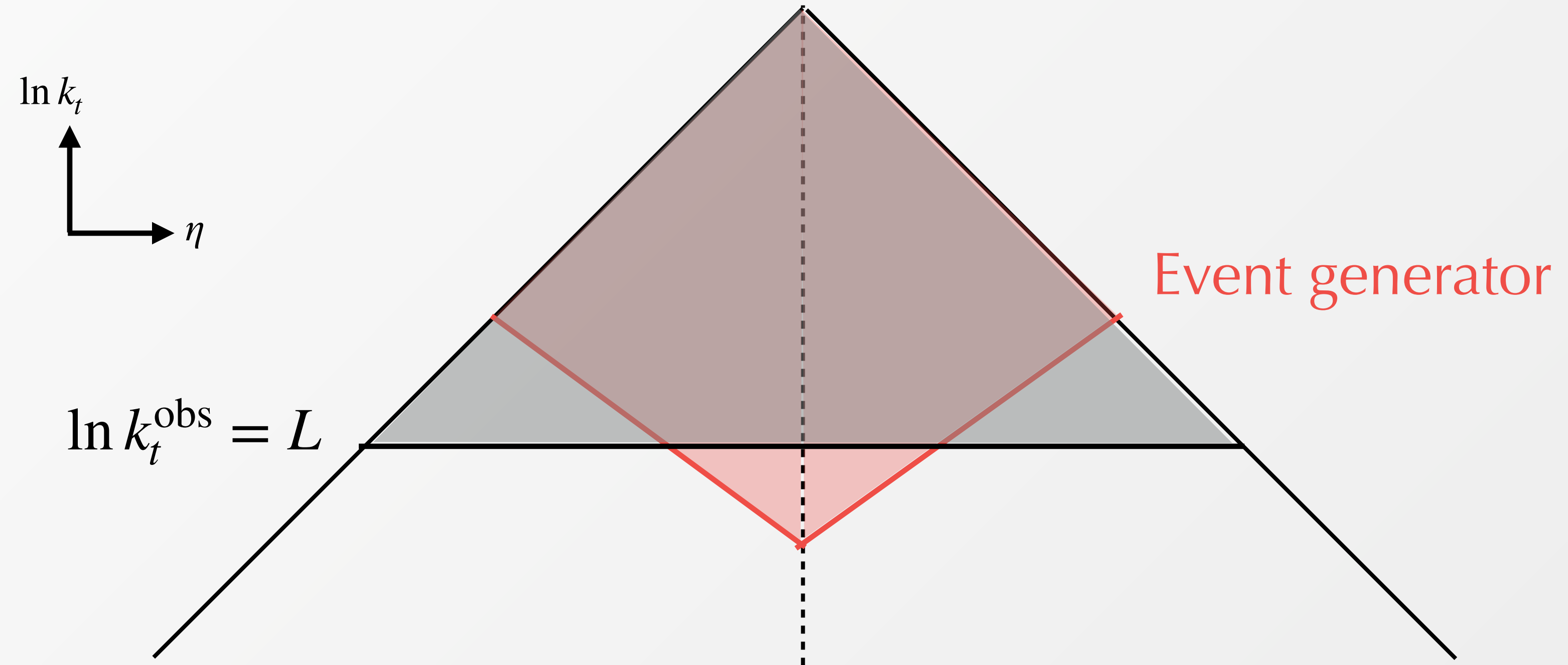


# Interplay with the parton shower

The event generator generates the hardest emission with an associated Sudakov suppression factor

Let's assume that the event generator is characterised by an resolution variable scaling as

$$O_{EG} \sim \frac{k_t}{Q} e^{-\beta_{EG} |\eta|}$$

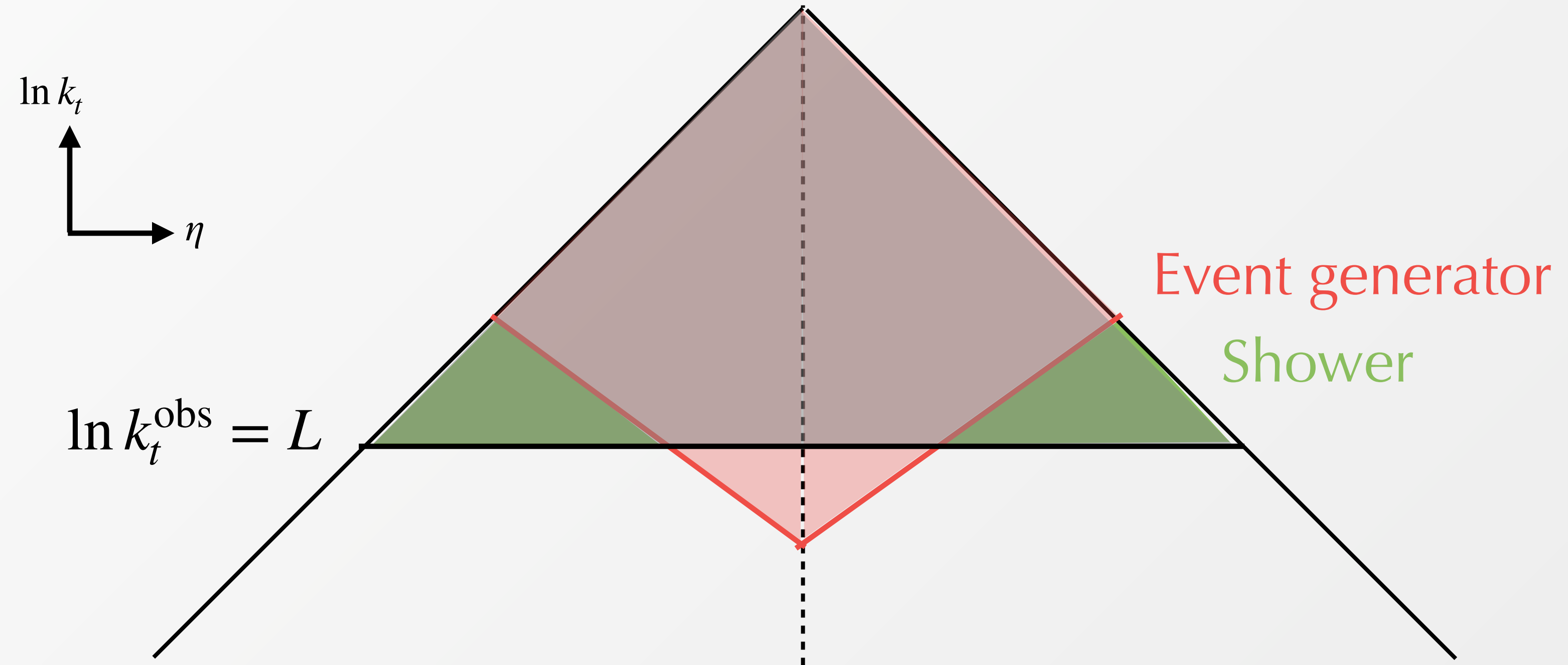


# Interplay with the parton shower

The remaining phase space which contributes to the probability  $\Sigma(O < e^L)$  is filled by the parton shower

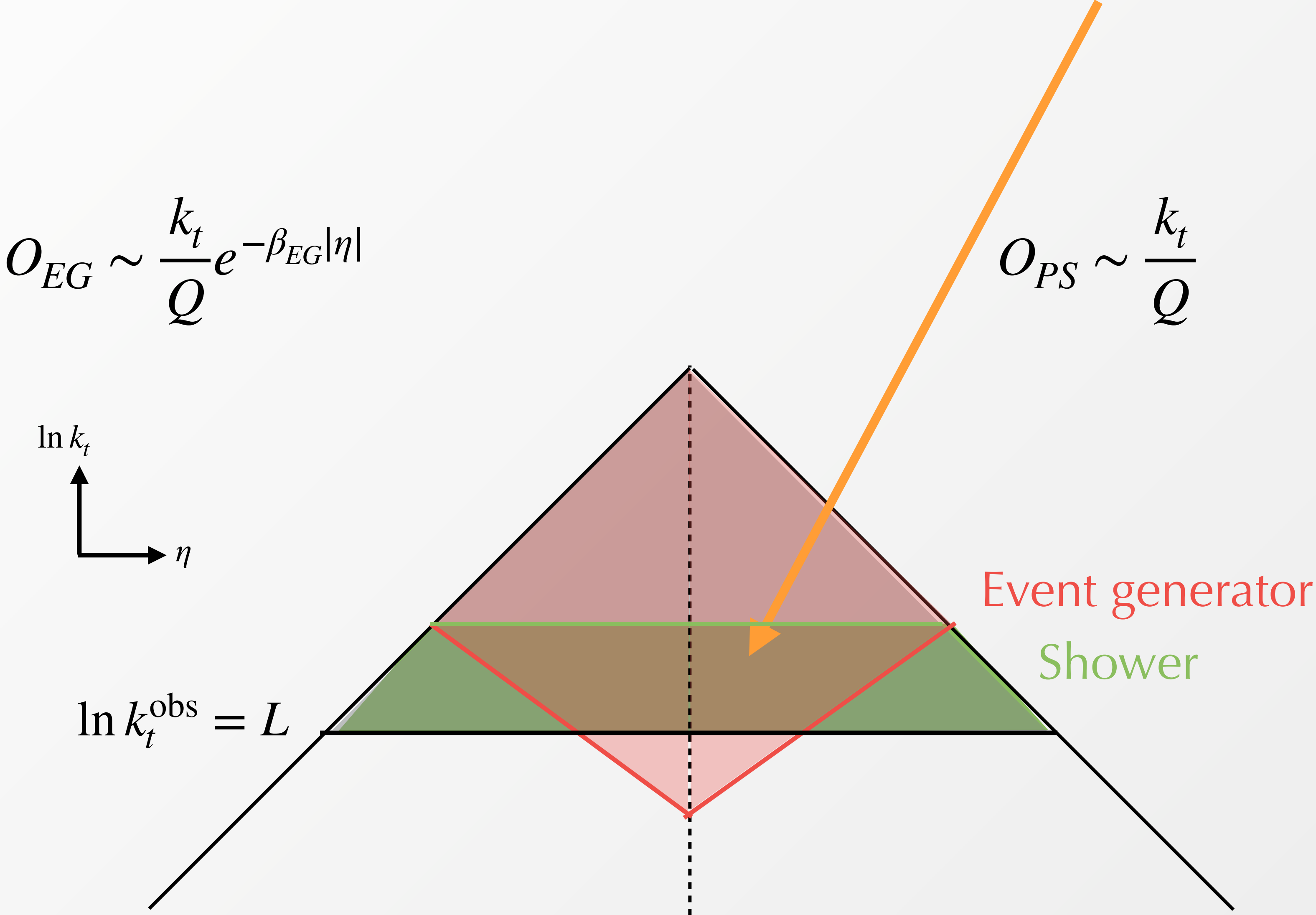
Here we again assume

$$O_{PS} \sim \frac{k_t}{Q} e^{-\beta_{PS}|\eta|}$$



# Interplay with the parton shower

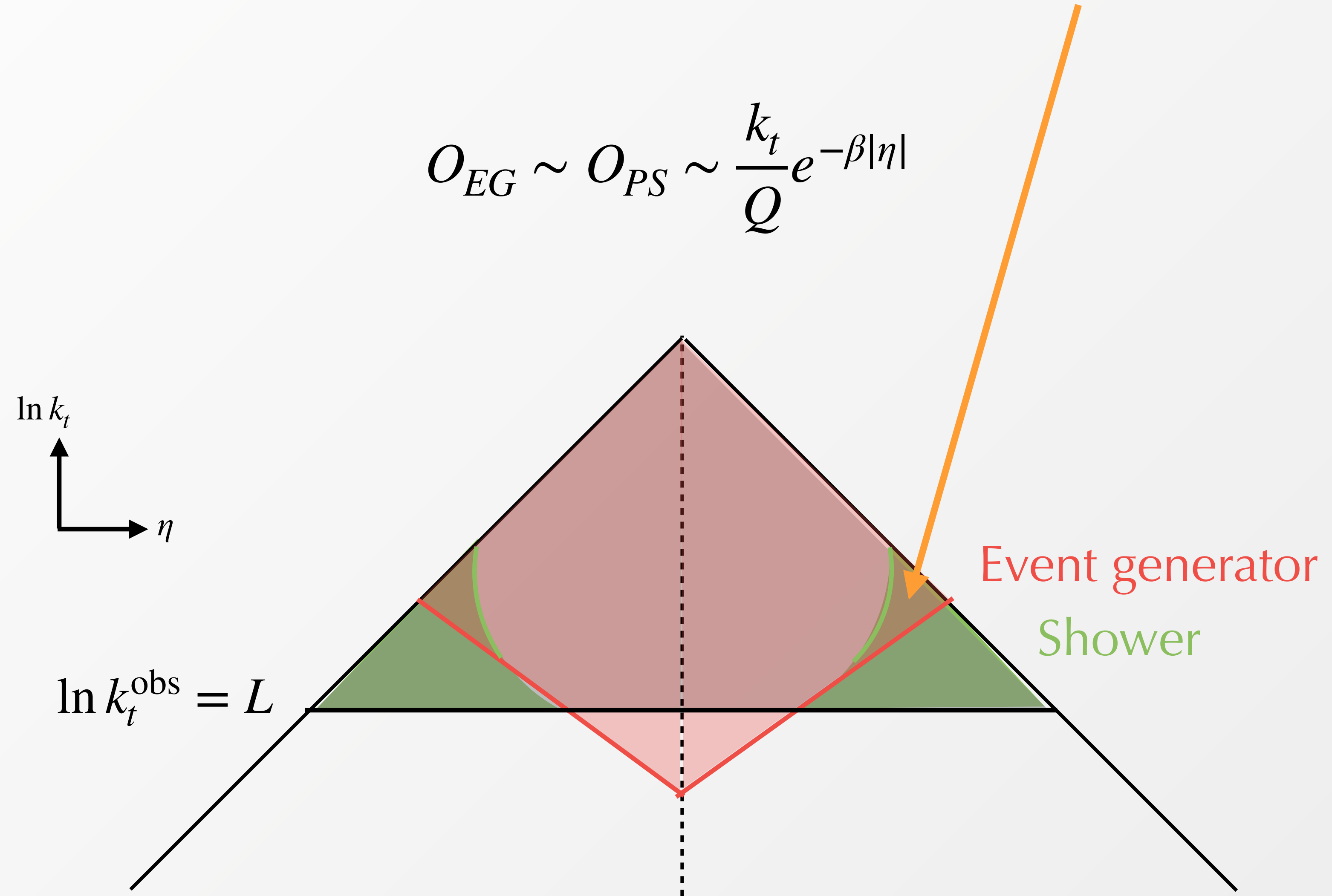
A mismatch between  $\beta_{PS}$  and  $\beta_{EG}$  breaks LL accuracy due to **double counting**



# Interplay with the parton shower

To achieve NLL (and beyond) accuracy after matching, in addition to have  $\beta_{PS} = \beta_{EG}$ , one must ensure the absence of contour mismatch in e.g. the hard-collinear region

$$O_{EG} \sim O_{PS} \sim \frac{k_t}{Q} e^{-\beta|\eta|}$$



## Interplay with the parton shower

At NNLO+PS the picture is more complex since the event generator takes care both of the **first** and **second hardest** emission, with the remaining emissions provided by the PS



## Interplay with the parton shower

At NNLO+PS the picture is more complex since the event generator takes care both of the **first** and **second hardest** emission, with the remaining emissions provided by the PS

- MiNNLO $_{q_T}$  (and GENEVA $_{p_T^{j,1}}$ ) allow for a **straightforward matching (at LL accuracy)** when ( $k_t$ -ordered) shower are employed, thanks to similarities between their resolution variables
- GENEVA $_{\mathcal{T}_0}$  (and GENEVA $_{q_T}$ ) resort to **truncated-vetoed shower** in the effort to preserve LL accuracy of the parton shower when matching with ( $k_t$ -ordered) showers
- GENEVA $_{q_T}$  with generalised jettiness variables is **claimed to be LL** accurate without the need to use truncated-vetoed shower; nevertheless, a deeper understanding of the interplay of the  $N$ -jettiness clustering procedure with the shower matching is required.
- MiNNLO $_{\mathcal{T}_0}$  **formally breaks LL accuracy** when matched to PYTHIA, as a change in the POWHEG mapping will be required to treat consistently the second emission

# Interplay with the parton shower

Formalisms based on transverse-momentum like observables ( $\beta = 0$ ) are favoured when matching with  $k_t$ -ordered showers as they facilitate the matching

Use of showers with a resolution variables with  $\beta \neq 0$  (e.g. DEDUCTOR) or angular ordered showers (e.g. HERWIG) would require additional care, **especially beyond LL**

**Too many handles** in LL accurate parton showers **make formal accuracy not so relevant practically** (predictions for 1-jet obs. can change significantly **simply by acting on the tune**)

These aspects **are now crucial** since (N)NLL-accurate (and beyond) parton showers for hadron collisions are becoming publicly available

# Phenomenology

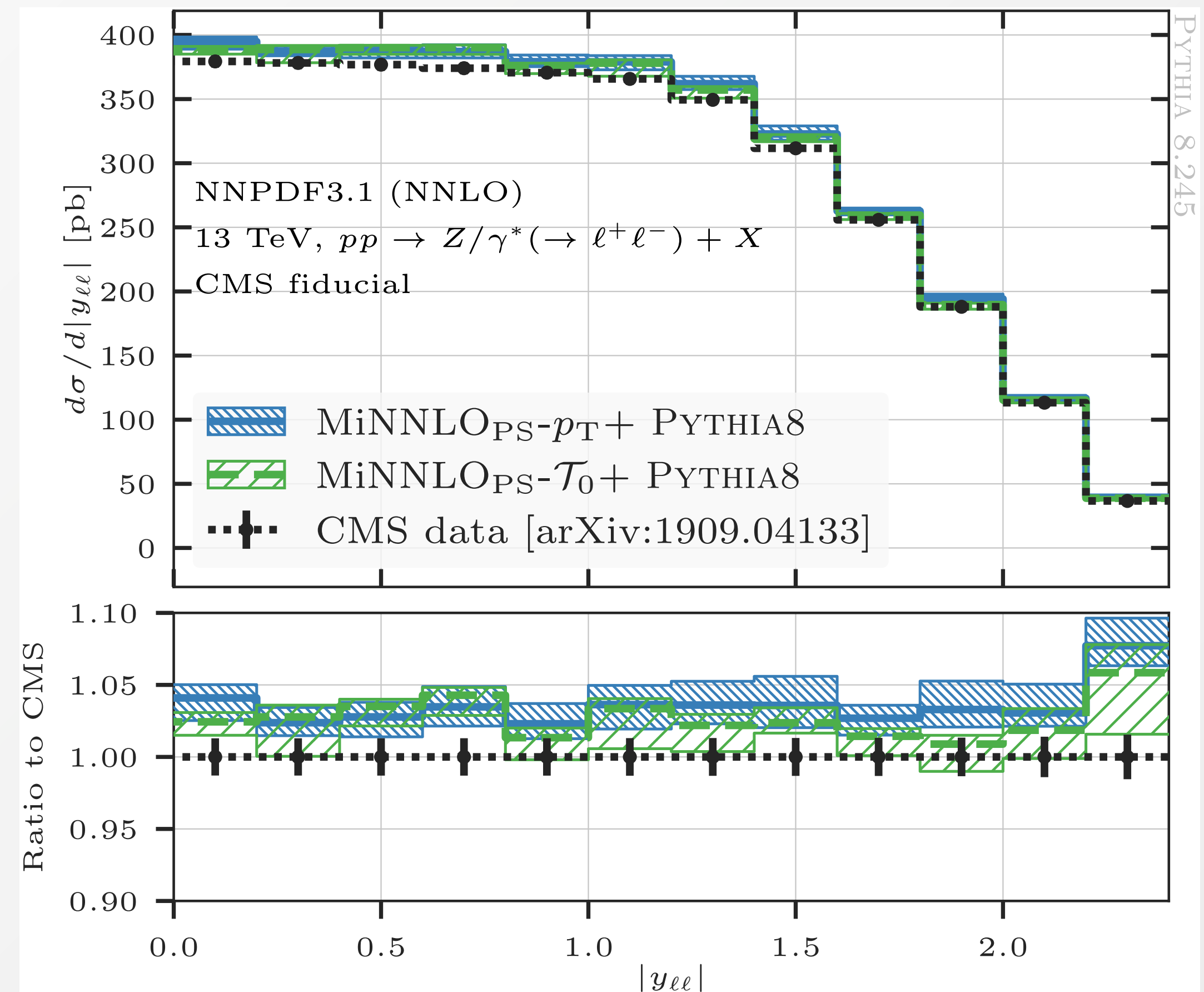
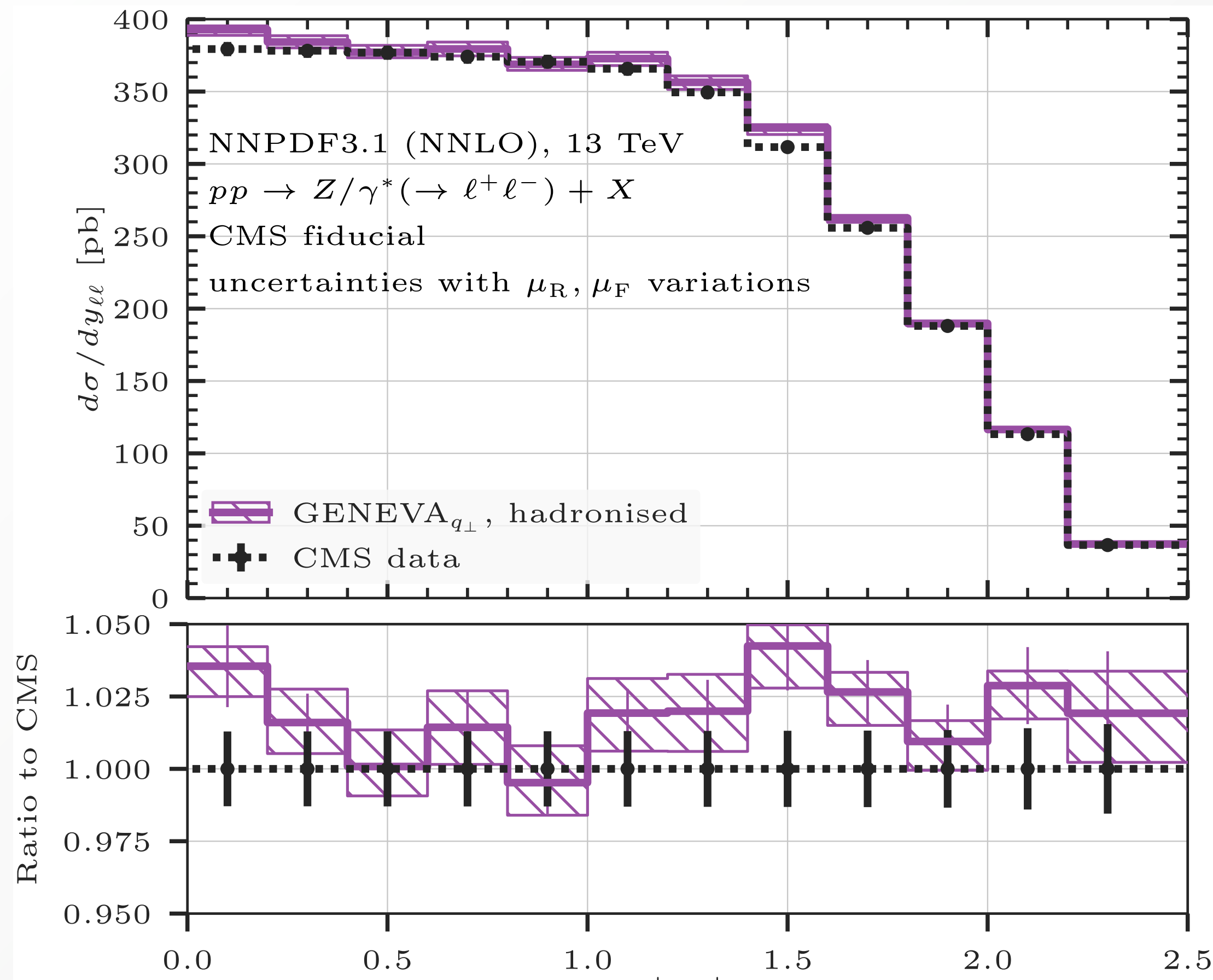
The choice of resolution variables is **in principle immaterial** to reach NNLO accuracy

However, its choice has important consequences

- Size of missing power corrections **(in the GENEVA method)**
- Ease of interface with the shower
- Overall description of **physical events** after matching and showering

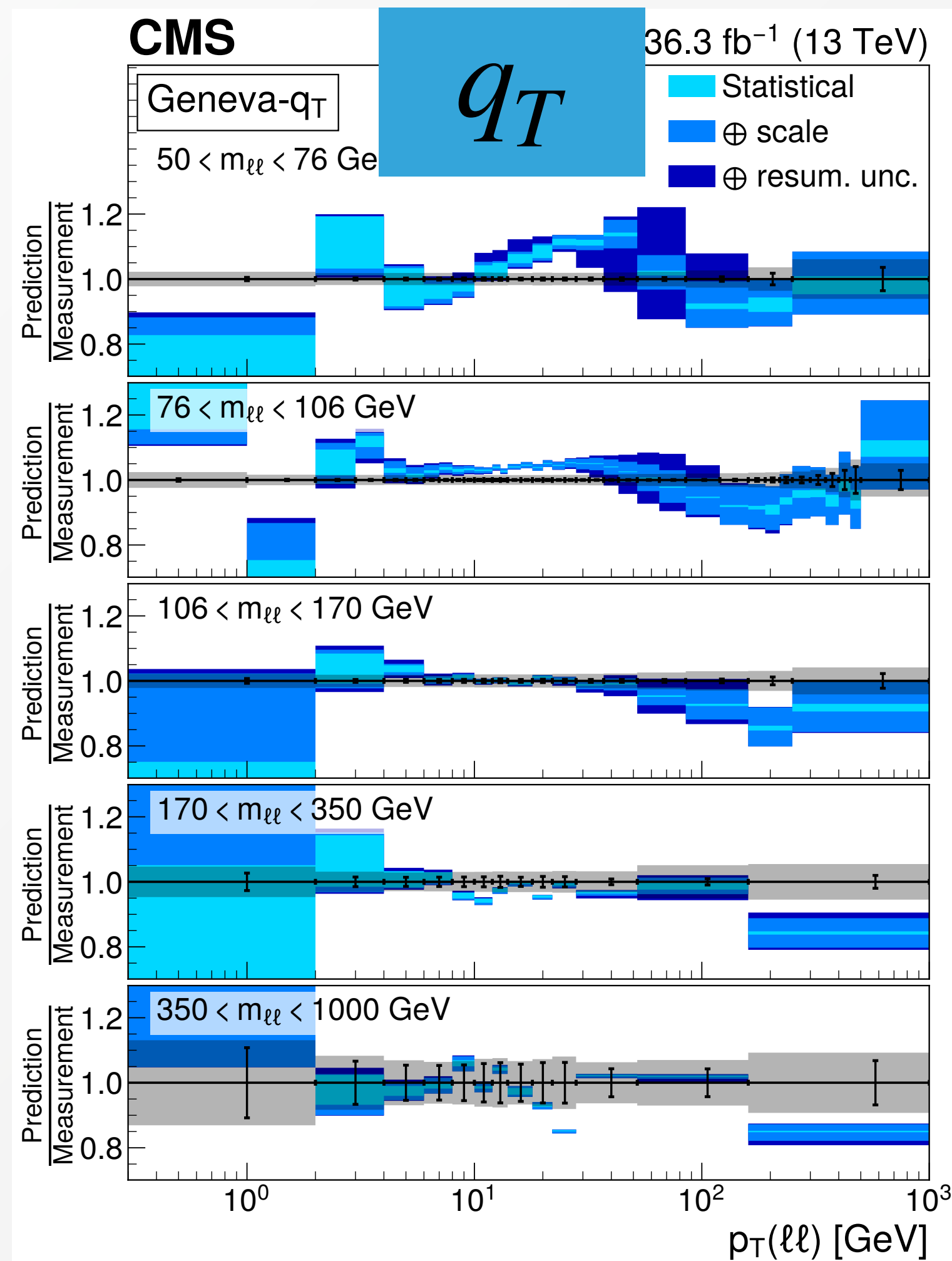
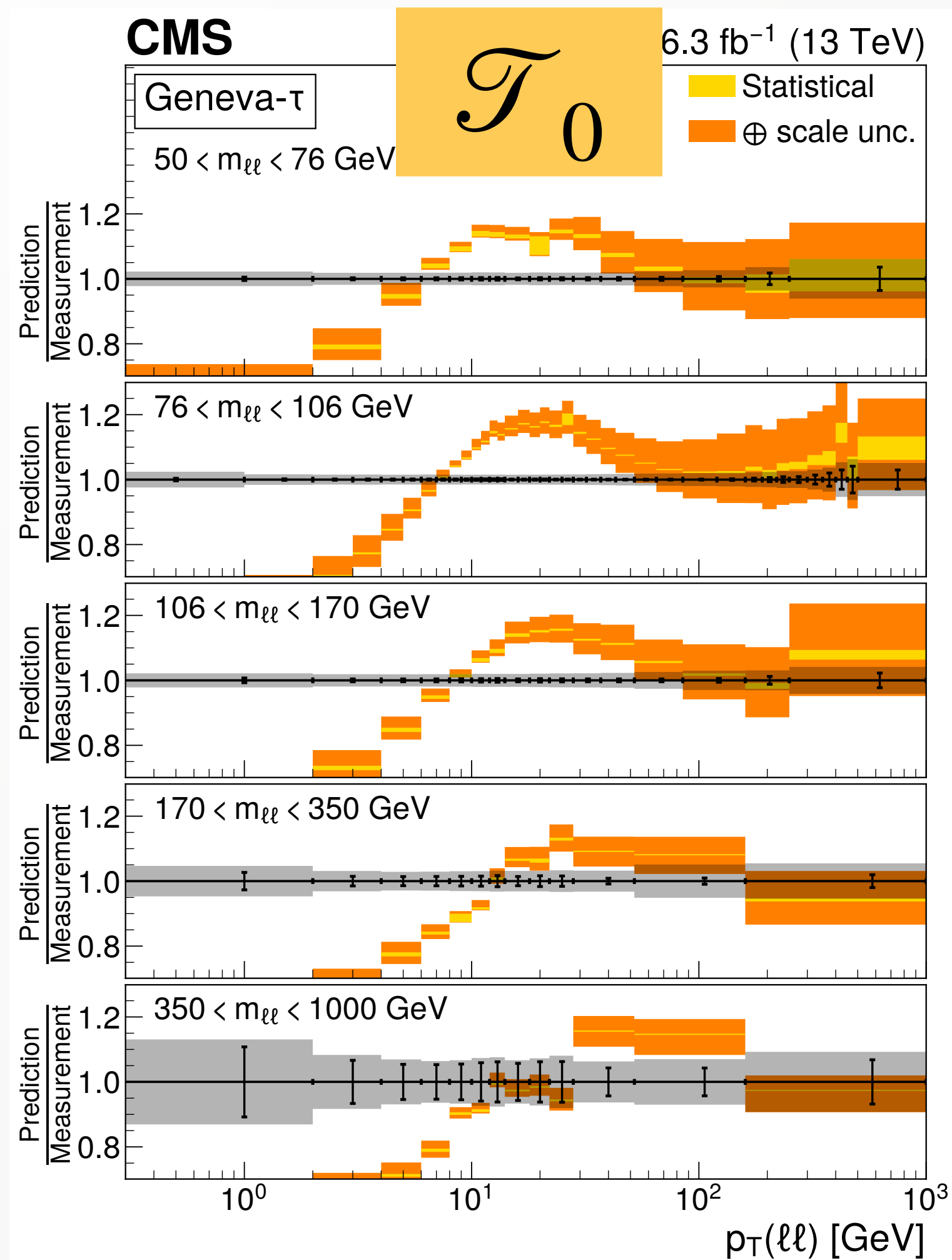
# Phenomenology: NNLO inclusive distributions

Results for NNLO inclusive observable should be (almost) independent on the resolution variable used



# Phenomenology: transverse observables

The situation is instead different for more differential observables, for which the details of the implementation and the **interplay with the parton shower** play an important role

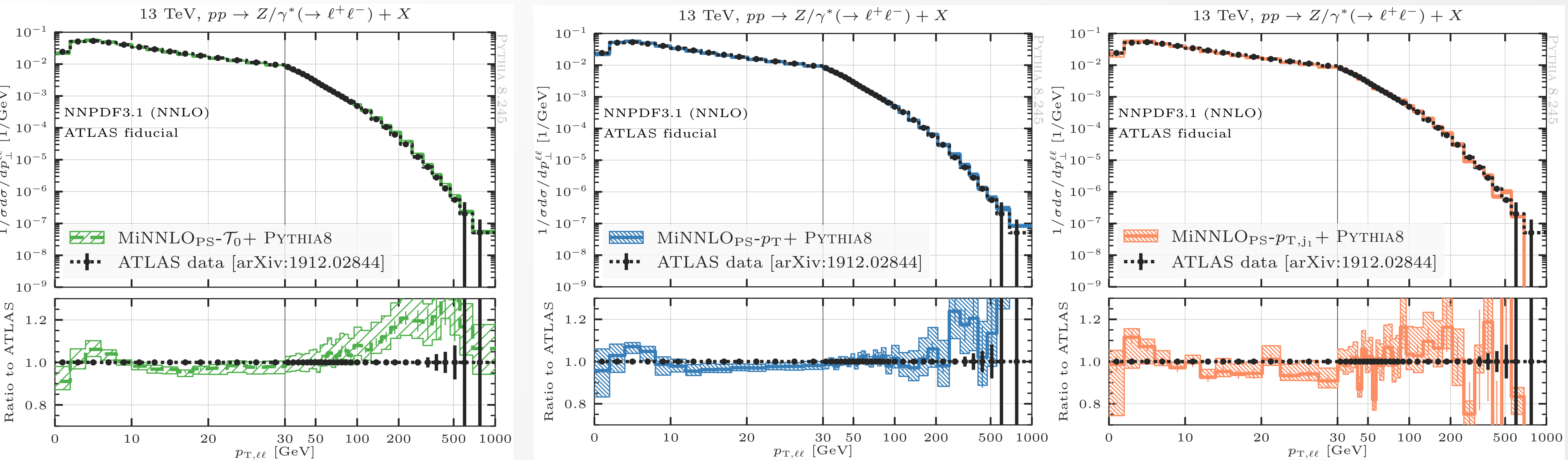


It would be interesting to see how GENEVA- $p_T^j$  performs (e.g. dependence on the jet radius), even more so since it features a different 1-2 jet separation variable

[CMS, 2205.04897]

# Phenomenology: transverse observables

The situation is instead different for more differential observables, for which the details of the implementation and the **interplay with the parton shower** play an important role



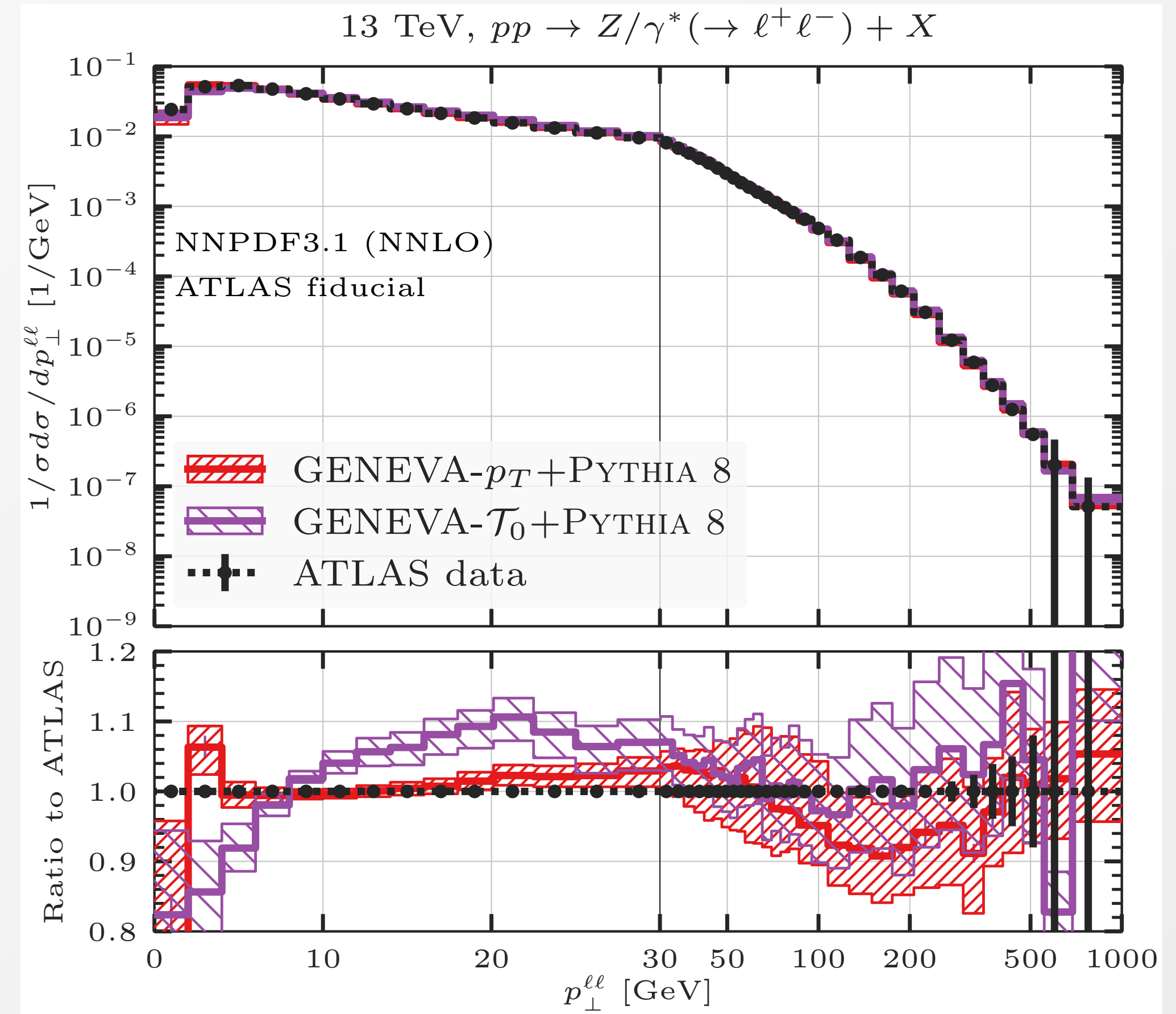
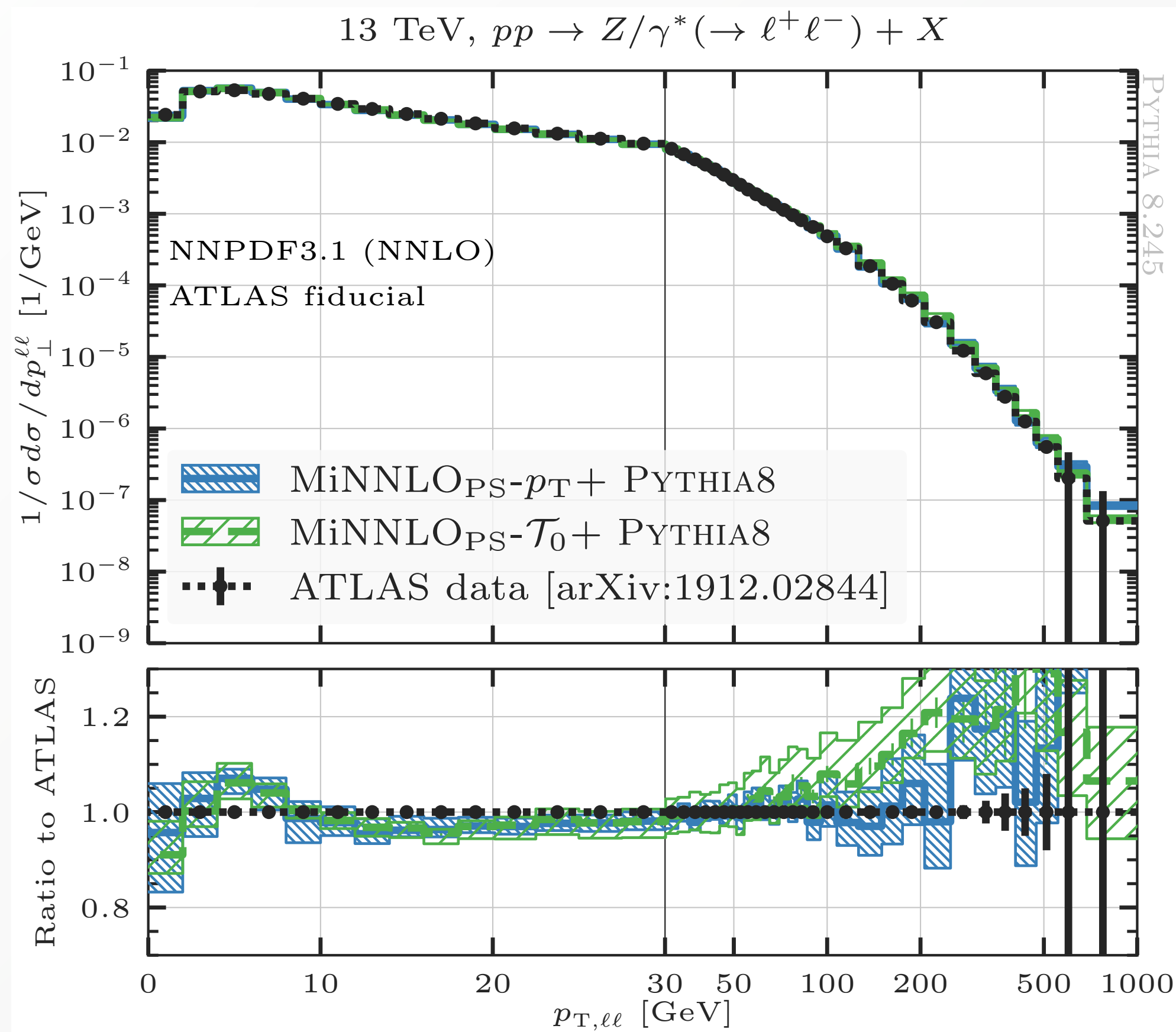
$\mathcal{T}_0$

$q_T$

$p_T^j$

# Phenomenology: transverse observables

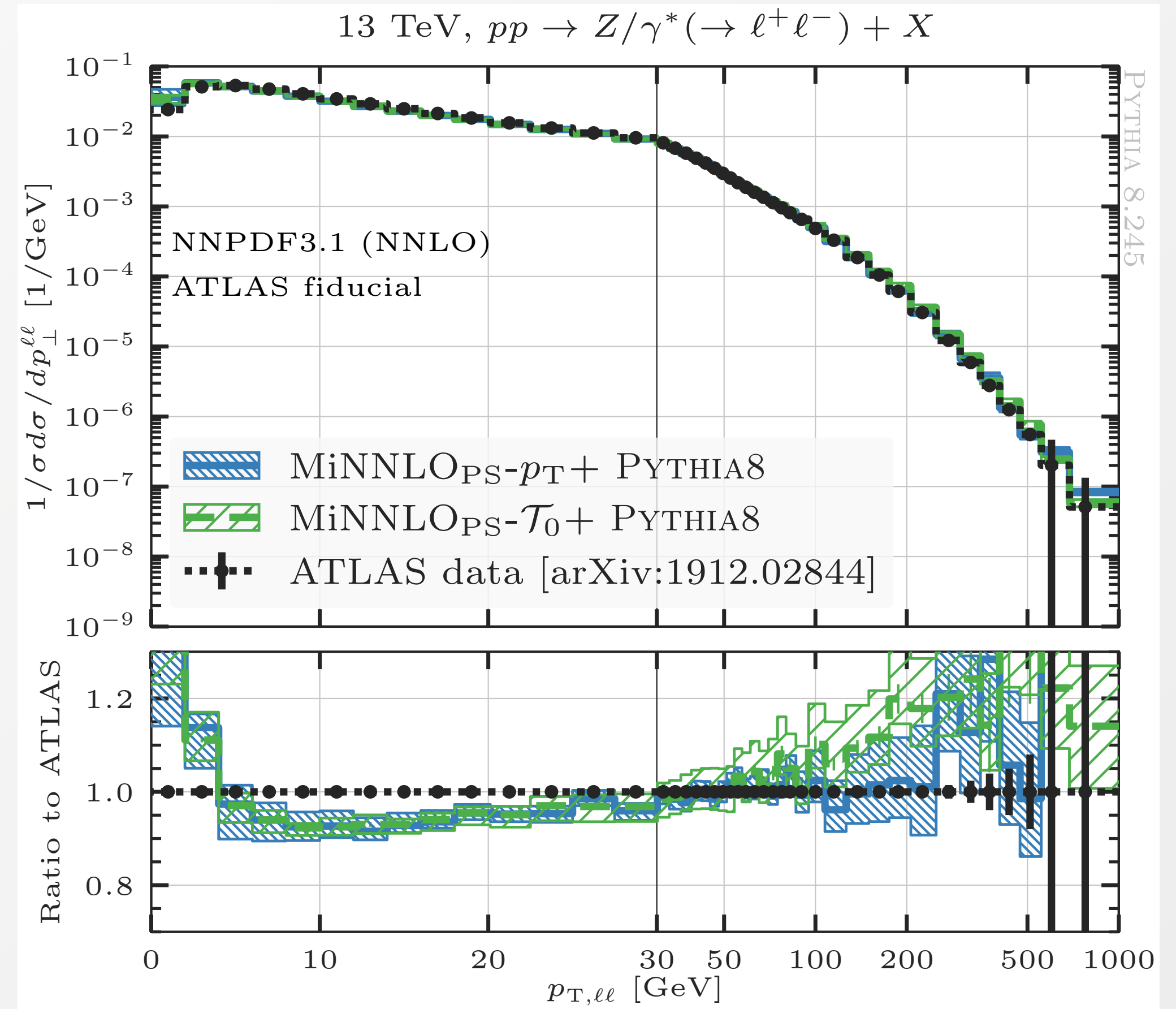
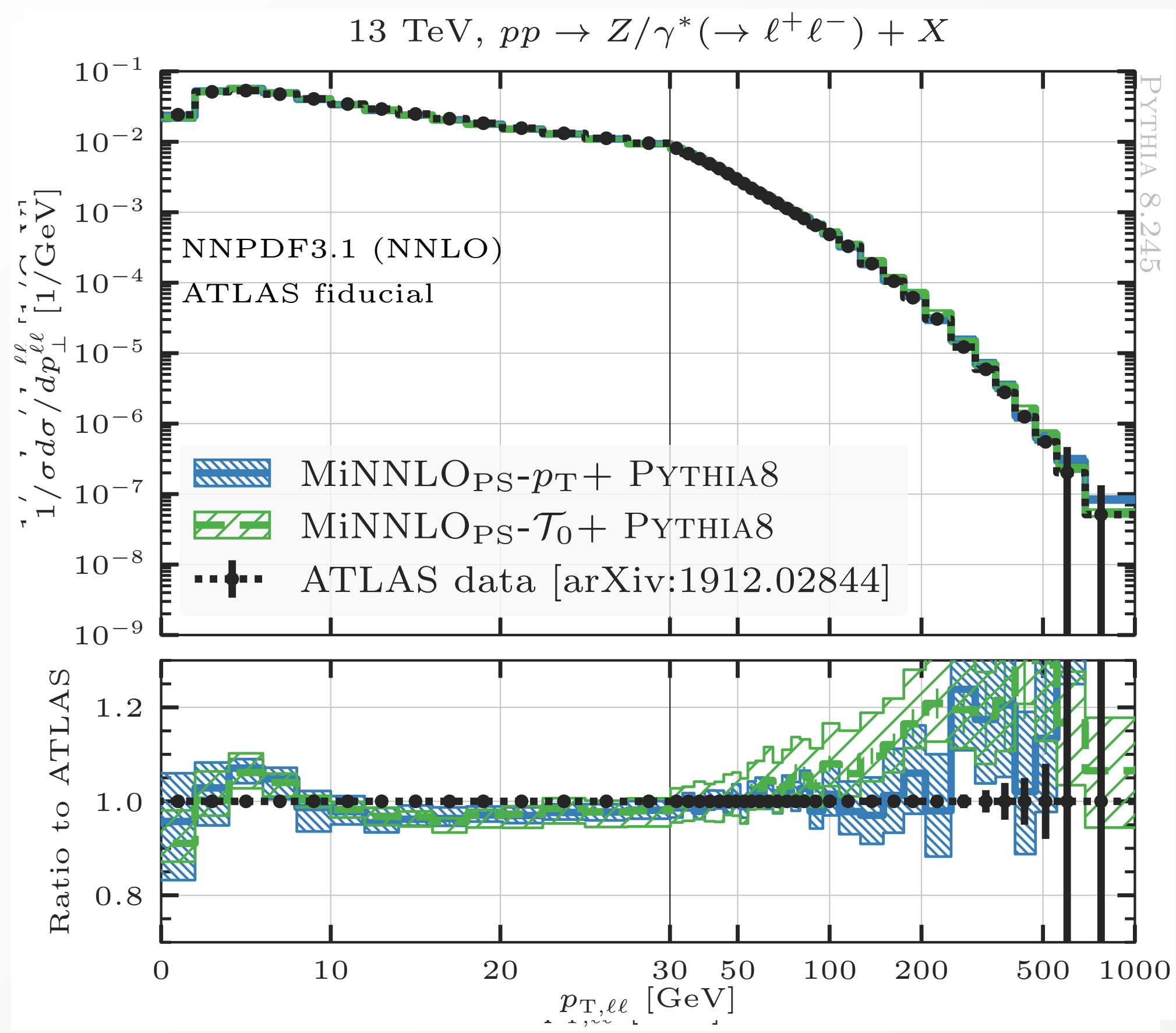
The situation is instead different for more differential observables, for which the details of the implementation and the **interplay with the parton shower** play an important role



Caveat: different tunes used for GENEVA and MiNNLO

# Phenomenology: transverse observables

The situation is instead different for more differential observables, for which the details of the implementation and the **interplay with the parton shower** play an important role

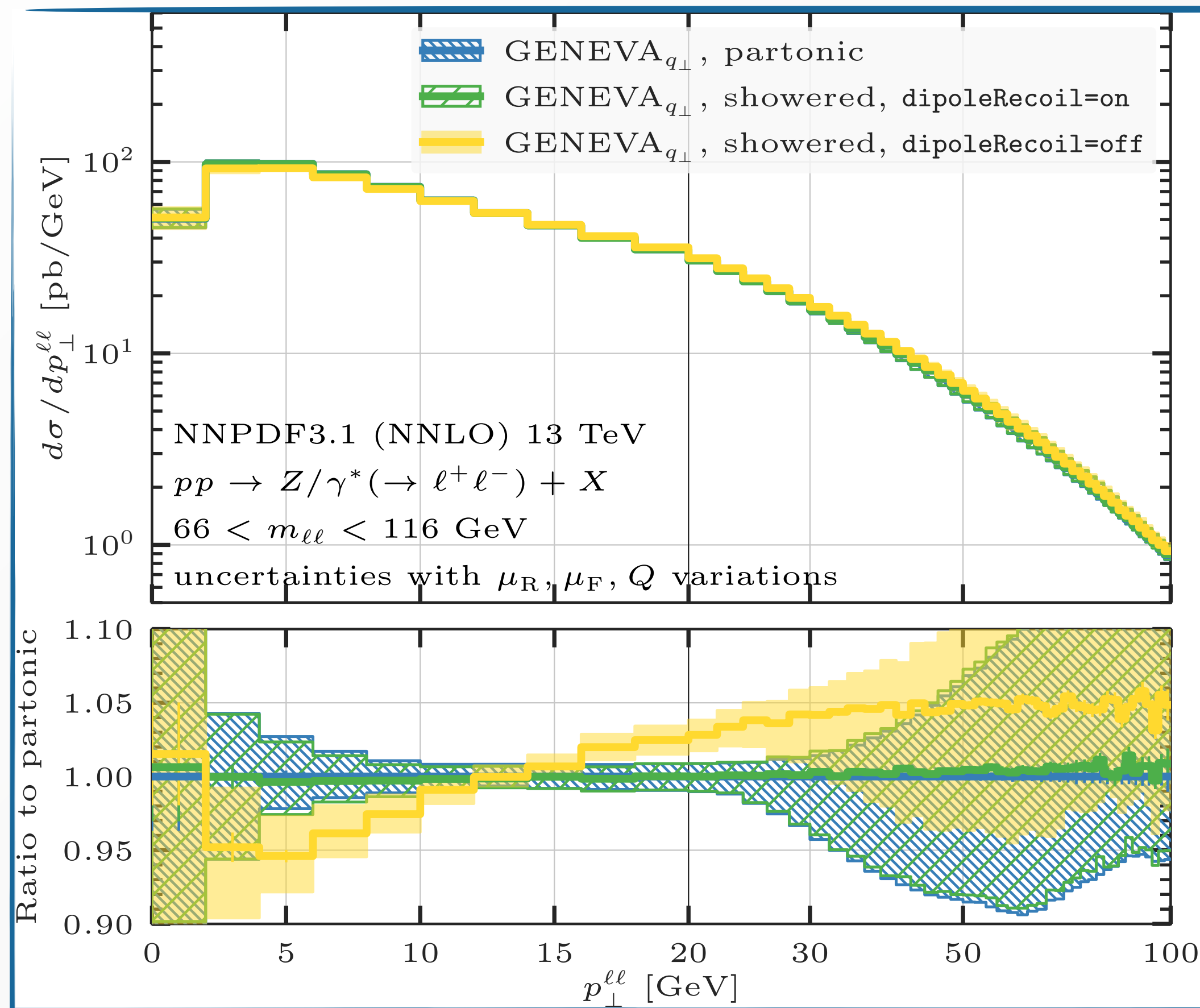


NNLO<sub>PS</sub>-optimised CMS tune

MonashStar tune

# Phenomenology: transverse observables

Analogue dependence on tune settings was observed in GENEVA

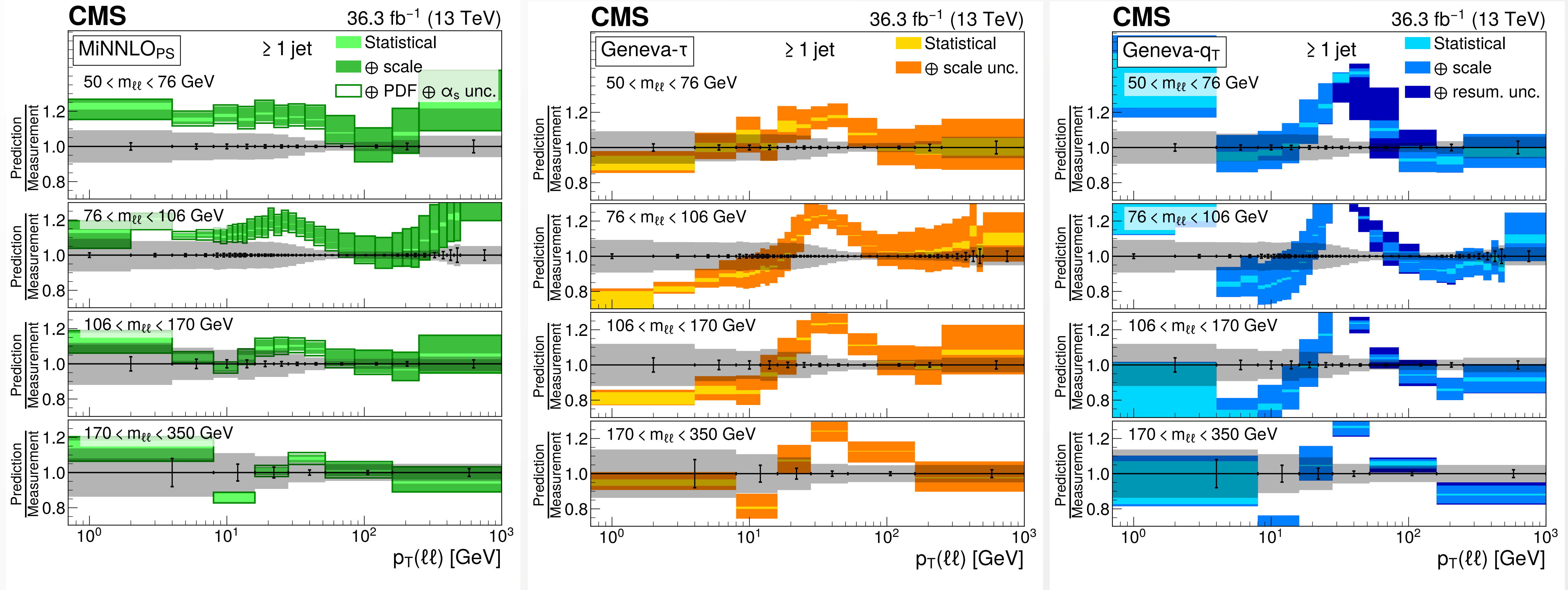


Non-negligible dependence on the recoil scheme of the shower, which can affect the transverse momentum spectrum at the few percent level (**besides affecting shower accuracy**)

Tunes obtained by comparing LO predictions to data are bound to **absorb higher order corrections into the tune parameters**

# Phenomenology: transverse observables

Although formally equivalent (and NLO accurate), the description of observables in the 1-jet phase space is also affected by the details of the implementation

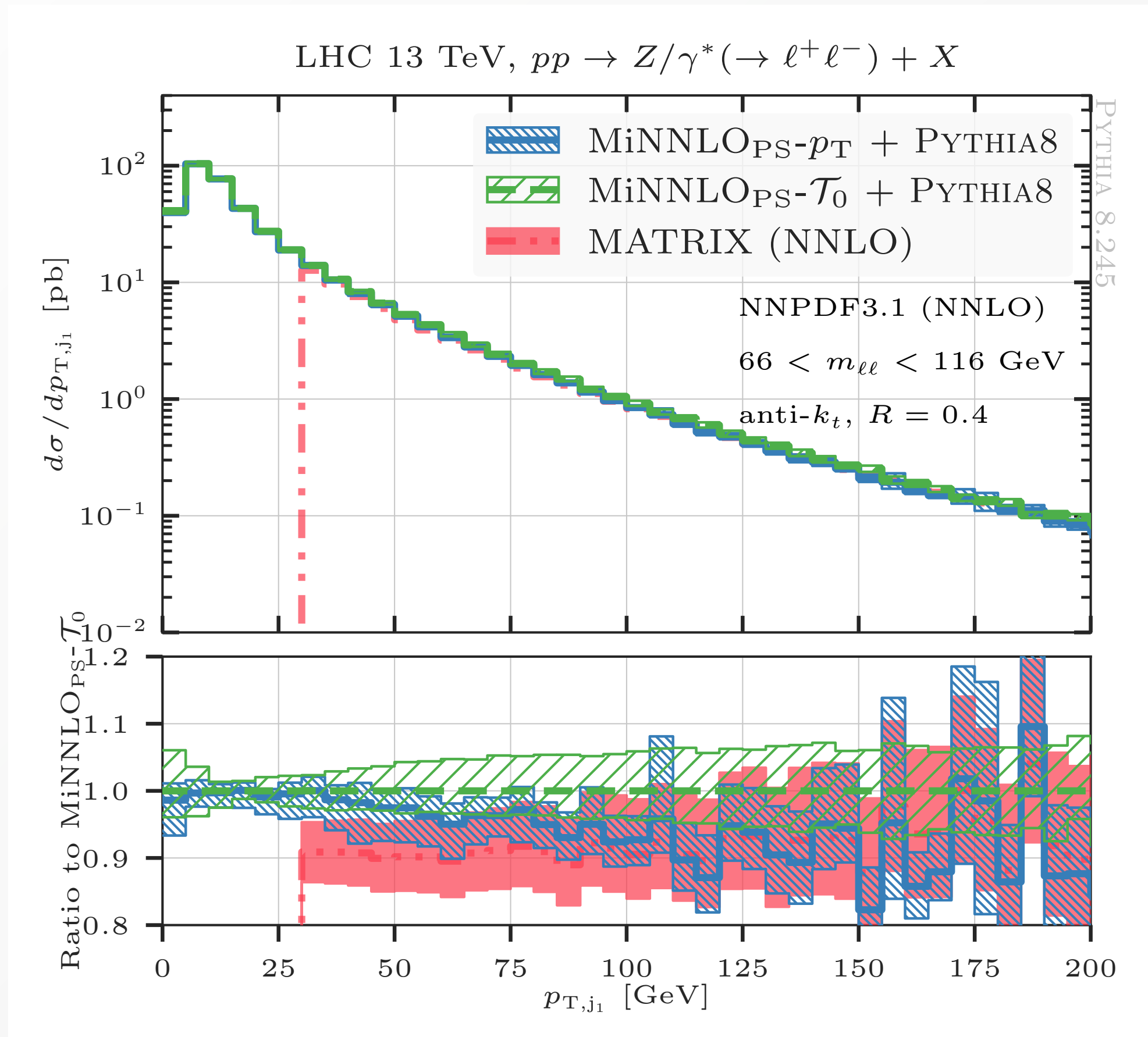


With what accuracy the different formalisms resum **Sudakov shoulder** logs?

# Phenomenology: transverse observables

Although formally equivalent (and NLO accurate), the description of observables in the 1-jet phase space is also affected by the details of the implementation

These differences are partially driven by how the (**formally higher order**) corrections re-distributed in the V+j phase space



$$\dots + \left( D(p_T) - D^{(1)}(p_T) - D^{(2)}(p_T) \right) \times \mathcal{P}(\Phi_{\text{FJ}})$$

$$\dots + \left( D(\mathcal{T}_0) - D^{(1)}(\mathcal{T}_0) - D^{(2)}(\mathcal{T}_0) \right) \times \mathcal{P}(\Phi_{\text{FJ}})$$

# Conclusion and some thoughts (1)

- A new generation of tools with **higher formal accuracy** are being developed, led by the advancement in the understanding of perturbative QCD
- Comparisons between different formalisms and alternative resolution variables lead to **challenging open questions** regarding the reliability of **current uncertainties** at NNLO+PS level, even for 'simple' candle processes such as Drell-Yan
- Will these differences persists when matching with parton showers with higher logarithmic accuracy?

## Conclusion and some thoughts (2)

The interplay with the shower is most likely the most delicate aspect to be addressed

- Despite efforts to improve on shower accuracy, very few studies have been performed in the context of NNLO+PS, **even at LL accuracy**
- Studies in simple environments (e.g.  $e^+e^-$ ) may be easier; however, **no NNLO ingredients for resolution variables besides 2-jettiness are available**
- Ensuring that shower accuracy is preserved far from trivial; difficult to envisage tests analogue to those performed in the context of parton showers
- Several strategies can be conceived: top-down (first achieve NNLO accuracy; then fix matching with the shower) or bottom-up (start from the shower; build up NNLO). Each has its advantages and limitations

## Conclusion and some thoughts (3)

Extension towards more complex processes with more than two coloured leg requires suitable resolution variables

- Currently much more limited choice of resolution variables, with  $N$ -jettines being the only candidate whose ingredients are known at NNLO for jet processes, and  $q_T$  the only option available for heavy quark production
- Although alternative variables have been proposed, a major bottleneck is the calculation of NNLO ingredients (jet, soft, beam functions)
- Concurrent progress in the resummation of multi-leg observables crucial (very few variables for multi-leg case known at NNLL or beyond)
- Still much work to be done if we aim to establish NNLO+NLLPS matching as a **novel standard of precision** for LHC processes!